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Abstract—This paper presents a method for studying the occurrence of loss of component scenarios in a power system using stochastic point process modeling. It can manage stationary and non-stationary failure and repair rates, common mode failures of any order, components with any number of operating states and diverse repair logistics. The failure and repair sequences in each maintenance zone of the power system are generated using Monte Carlo simulation. A list of possible loss of component scenarios is therefore not required as in other methods. Loss of component scenarios are studied in terms of probability of occurrence, expected frequency and mean duration. By means of two examples, it is shown how the proposed method is a more flexible modeling tool than those based on Markov chains.

Index Terms—Interconnected power systems, power system reliability, point processes, queuing analysis.

I. INTRODUCTION

A loss of component scenario (LCS) is the situation where a system with \( n \) components has \( x \) components out of service due to planned or unplanned events. It is denoted as a \( n-x \) LCS and it is called a LCS of order \( x \). Only failures which force components to the unavailable state or derate their capacity are considered here as unplanned events. Planned events are not considered, because they can be programmed for a time when system reliability is not affected. A component is defined as an item of primary plant of the system (such as a transmission line, transformer or generating unit.) or an item of secondary plant (such as auxiliary services systems, protective relays). Thus, component failure models thus include failures of both types. If each component has two states, “available” and “unavailable”, the state space of operating states of a system with \( n \) components has dimension \( 2^n \). As example, consider the system shown in Fig. 1. It has 7 components, 128 operating states, seven \( n-1 \) LCS, twenty-two \( n-2 \) LCS, etc.

The term operating state refers here to a LCS and does not qualify the system operating condition in terms such as “Normal”, “Alert”, or “Emergency”, as used in the context of power system security [1], [2].

Because every power system has many components, the dimension of the space of operating states is huge; hence, every study on the occurrence of LCS is a cumbersome task, no matter the assessment method applied. This paper presents a method based on Stochastic Point Process (SPP) modeling which offers great modeling flexibility but requires a lot of computational resources.
II. PROBLEM STATEMENT

A. Objective of the study

The aim is to measure the occurrence of LCS by means of the probability of occurrence $P_{n-x}$, the expected frequency $F_{n-x}$ and the mean duration $D_{n-x}$. Although this objective may appear similar to the one of the reliability assessment technique called “state enumeration” [3]-[4], the difference is that in the present study the indices are determined globally for the set of operating states that belong to a given LCS order and not for each of them separately. For example, considering the system shown in Fig. 1, LCS indices are calculated for the $n-2$ LCS order, not individually for the twenty-two operating states that belong to this order.

B. Traditional modeling for reliability studies

This kind of modeling is presented in many references, for example [3]-[7]. For simplicity, in the following discussion, only the basic component reliability model of two operating states shown in Fig. 2 is considered. However, the concepts explained here extends to models with any number of states.

Fig. 2 Two state component reliability model

This model is defined by means of a failure rate $\lambda$ and a repair rate $\mu$, which can be estimated from operating records that cover a period $T$ in the following way:

\[
\lambda = \frac{1}{\text{ttf}} = n_f/((T - \sum_{r=1}^{k} ttr_r)) \quad (1)
\]

\[
\mu = \frac{1}{\text{ttr}} = n_r/((T - \sum_{f=1}^{k} ttf_f)) \quad (2)
\]

Where, ttf , $n_f$ , ttr and $n_r$ denote respectively the time to failure, the number of failures, the time to repair and the number of repairs. Over-lined symbols, such as $\bar{x}$, denote the statistical mean of the variable. Two important component reliability indices are the availability ($A$) and the unavailability ($U = \overline{A}$), which can be estimated from operating records in the following way:

\[
A = P_i(\infty) = \mu / (\lambda + \mu) = \frac{\sum_{j=1}^{n} ttf_j}{T} \quad (3)
\]

\[
U = P_i(\infty) = \lambda / (\lambda + \mu) = \frac{\sum_{j=1}^{n} ttr_j}{T} \quad (4)
\]

Where $P_i(\infty)$ denotes the probability of finding state $i$ in the long run. Constant failure and repair rates mean the failure and repair processes are stationary and hence, in the long run, $A$ and $U$ tend towards the estimates (3) and (4) independently of the probability distribution of ttf and ttr.

If the distributions of ttf and ttr of all components are exponential, the LCS study can be solved using the continuous Markov chain method. Fig. 3 shows a Markov chain representation for a system with $n$ components.

Fig. 3 Markov chain diagram for the state space of system operating states

The mathematical formulation of this method for a system of $n$ components is the set of ordinary differential equations:

\[
[\dot{P}]' = [M]' [P]' \quad (5)
\]

Where $[P]$ is a vector with the probabilities of the $n \times n$ states and $[M]$ is the matrix of transition rates between states.

If $k$ operating states are LCS or order $x$ the probability of occurrence of LCS of this order is:

\[
P_{n-x} = \sum_{j=1}^{k} P_j(\infty) \quad (6)
\]

The expected frequency and the mean duration of LCS of order $x$ are computed as:

\[
F_{n-x} = P_{n-x} \times \sum_{j=1}^{k} h \quad (7)
\]

\[
D_{n-x} = 1 / \sum_{j=1}^{k} h \quad (8)
\]

Where \( \sum h \) is the sum of the rates of all the transitions leaving states belonging to an LCS of a particular order, i.e. transitions across the dashed lines in Fig. 3.

If the failure and repair processes of all components are independent, it is not necessary to solve (5) first to apply (6). $P_{n-x}$ can be obtained by applying the formula of simultaneous occurrence of independent events ($E_i$):

\[
P([E_1, E_2, \ldots, E_k] = \prod_{j=1}^{k} P[E_j] \quad (9)
\]

Where $E_i$ denotes the operating state ($A_i$ or $\overline{A_i}$) of component $i$.

If all component failure and repair processes are stationary and independent, (9) is valid no matter the probability distributions used to represent these processes.

It is important to observe that (9) does not apply for the case where common failure modes exist and its extension to include them is very complicated. In this case, common...
failure modes have to be included in \([M]\) which can then be solved by applying (5). It is also important to point out that previous to the application of (5) or (9) a list of all possible operating states has to be elaborated and this is not an easy task.

C. What do the assumptions of traditional modeling imply?

1) Stationary failure and repair processes

Stationarity of failure and repair processes has been a common assumption in the field of power system reliability. However, the validity of this assumption must be examined carefully, especially for the failures. Because of component aging and lack of preventive maintenance, two conditions currently present in many power systems all over the world, component failure rates may not be constant but rather increasing functions of time.

On the other hand, a constant repair rate means that the performance of repair teams is not affected by internal or external factors. However, in real life, crew performance is affected by external factors such as weather and traffic, as well as by internal factors such as available tools and personnel skills.

Non-constant failure and repair rates also mean that the study of LCS cannot be done by applying (3) to (9). One way to overcome this problem is to introduce time varying rates into \([M]\) to solve (5), i.e. by making it into a non-homogeneous Markov chain process. However, this kind of process has problems with the adjustment of the operating times and with tractability for some types of time-varying rates [8]. Even, if the limitations of the non-homogeneous Markov chain process were considered unimportant, this kind of modeling has another drawback that is discussed next.

2) Independent component repair processes

It has also been a common practice in the field of power system reliability to include repairs as part of component models, by using the model shown in Fig. 1; this approach means that:

1. The repair process of each component is independent of the repair processes of other components.
2. Repair resources are unlimited because every time a component fails it is immediately repaired. This is equivalent to assuming that each component has a dedicated repair team.

Obviously, these assumptions are not realistic. For maintenance activities, a power system is typically split into maintenance zones which are assigned to repair teams, as shown in Fig 4. Repair resources are limited because crews have to fix all failed components located in their zone. Hence, some failures have to wait until the ones that occurred first are repaired. Thus, repairs of component located in a given maintenance zone are not independent. A proper way to represent the repair process performed in each maintenance zone of the power system is to use the queuing system shown in Fig. 5. For this system, the following needs to be defined:

- Input process: The failure process generated by all components located in the zone.
- Service process: The SPP that represents the repair team activities and is used to generate the times to repair.
- Output process: The SPP of the repairs performed by crews.

D. Proposal

To improve the modeling flexibility of the LCS study, it is proposed to represent the component failure processes using an SPP and the repair process of each maintenance zone of the power system using concepts from queuing theory.

III. STOCHASTIC POINT PROCESSES

A. General Concepts

A SPP is a random process in which the number of events \(N\) that occur in a period \(\Delta t\) is counted, with the condition that one and only one event can occur at every instant. Fig. 6 gives a pictorial representation of a SPP; \(x_i\) denotes an inter-arrival interval and \(t_i\) an arrival time. If the date when the observation of the process started is taken as reference, \(\Delta t = t - 0\), and only \(t\) appears in the equations that describe the process.

- Fig. 6. The concept of SPP
The mathematical model of a SPP is defined by the intensity function $\lambda(t)$:

$$\lambda(t) = dE[N(t)]/dt$$  \hspace{1cm} (10)

A SPP has positive tendency if event arrivals increase with time (inter-arrival intervals decrease), negative tendency if event arrivals decrease with time (inter-arrival intervals increase) and zero tendency if event arrival or inter-arrival intervals do not show a pattern of increase or decrease.

A SPP without tendency is stationary or time-homogeneous. Homogeneity means inter-arrival intervals are independent and identically distributed; hence, events that occur are independent. The opposite is true for a SPP with tendency.

The parameter $\lambda(t)$ controls the tendency in the mathematical model of a SPP. Fig. 7 shows a basic classification of SPP models.

A RP is defined by means of the distribution of the inter-arrival times. The most famous RP is the exponential one commonly called Homogeneous Poisson Process (HPP). The general procedure for fitting a SPP model to a data sample is presented in [9], [10].

B. How to generate samples from SPP models

1) Renewal Processes
   1. Let $t_0 = 0$.
   2. Generate a uniform random number $U_i$.
   3. Get an inter-arrival interval $x_i = F^{-1}(U_i)$ using the probability distribution function of the inter arrival intervals.
   4. The arrival time is $t_i = t_{i-1} + x_i$.
   5. Go to step 2 until the stopping rule is reached: a given number of events or a sample period $T$.

2) Non-Homogeneous Poisson Processes (NHPP) [11]
   1. Generate a sequence of $n$ arrival times from an HPP with intensity function $\lambda = 1.0$, which covers the sample period $T$. These times are called $t_1, t_2, \cdots, t_n$.
   2. Find the inverse function of the mean cumulative number of events of the NHPP under study ($\Lambda^{-1}$).
   3. Calculate the arrival times of the NHPP as $t_i = \Lambda^{-1}(U_i)$.
   4. Calculate the sequence of $x$.

As pointed out by Law and Kelton [11], the application of this algorithm depends on how easy the inversion of $\Lambda$ is.

IV. PROPOSED METHOD

A. Failure Process Modeling

1) Components with two operating states
   The failure process of a two state component is represented by means of a SPP model.

2) Component with more than two operating states
   Some people believe components with more than two operating states cannot be represented by SPP. This is not true. In this case, the component is represented by one SPP model for each derated operating state.

   For example, consider the Markov chain model for a power transformer with a forced cooling system shown to the left in Fig. 8. $\lambda_1(t)$ is the forced failure rate and $\lambda_2(t)$ the failure rate of the forced cooling system. In this case, the failure process of the power transformer is represented by two independent SPP as shown to the right in Fig. 8.

   Fig. 8. SPP modeling for a component with three operating states

3) Common mode failures
   Each common failure mode is represented by means of a SPP model. This means, common mode failures have to be separated from other types of component failures. Because the definition of SPP imposes that only one event can occur at any instant, common mode failures are considered point events with a given number of component disconnections are associated.

4) Longitudinal components
   A longitudinal component can be located in several maintenance zones. To properly manage this, two modeling approaches are considered:
   - To split the longitudinal component in as many subcomponents as maintenance zones it belongs. This means failure data has to be classified by maintenance zone in order to fit a SPP failure model for each subcomponent. In this case, the failure of any subcomponent produces a common mode outage on the other ones.
   - To use a single failure SPP failure model and to associate to it a distribution model $F_L(L)$ for the distance $L$ from a component terminal to the failure point. Every time a component failure is generated using the SPP failure
model of the component, a distance to failure $L$ is generated using $F_0(L)$; thus, $L$ gives information about the maintenance zone in which the failure is located.

**B. Repair Process Modeling**

The alternatives for defining the repair service model are:

- To use a RP or a NHPP. See [12] for details of the procedure for fitting a SPP model to samples of repair data.
- To generate a repair time in accordance to the type of equipment to be repaired. In this case, the repair service SPP is non-homogeneous in the sense that repair times sequences are not generated using a single distribution.

The queuing system in each maintenance zone of the power system is described in Kendall’s notation as [13]:

\[ G/G/m/\infty/FCFS \]

The first and second “G” indicate that input and service processes are general SPP (RP or NHPP). $m$, $\infty$, and $FCFS$ indicate, respectively, the number of crews, the system capacity (all failures have to be repaired) and the queuing discipline (First Come First Served).

**C. Algorithm of the proposed method**

A sequential Monte Carlo simulation [14] is used to generate the sequences of failures and repairs in each maintenance zone of the power system.

The steps of this procedure are:

1. Generate for the period of study $T$ of one or more years the failure processes of all the components, i.e., generate for each component and common failure mode a sequence of times to failure in accordance with its SPP model and using the algorithms given in section III.B.
2. For each maintenance zone of the power system, superpose the failure processes of all components located in that zone.
3. For each maintenance zone of the power system, generate the repair process using the SPP repair service model, i.e., for each failure that occurs in a given zone, generate a time to repair in accordance with the SPP repair service model for that zone, using the algorithms given in section III.B.
4. Failures and repairs define sub intervals of different LCS order as shown in Fig. 9. Each failure $f_j$ has an associated repair $r_j$ that lasts $tr_j$, but some repairs are delayed due to congestion in the repair system, i.e. some failures have to wait to be repaired until the repairs of other components that failed before are finished.
5. For each LCS order count the number of subintervals $N_{(n-x)}$ and compute the sum $d_{(n-x)}$ of these intervals.
6. Repeat steps 1 to 4 for $N$ samples
7. Compute for each sub-period $k$ (month, semester, year, etc.) of $T$ the LCS indices in the following way:

\[ P_{n-x} = \frac{\sum_{j=1}^{N} d_{(n-x)}}{N \times T} \]  

\[ D_{n-x} = \sum_{j=1}^{N} d_{(n-x)} / (N \times T) \]  

\[ F_{n-x} = \sum_{j=1}^{N} N_{(n-x)} / (N \times T) \]

Contrary to (5) and (9), the proposed method does not require a table of possible LCS.

\[ D_{n-x} = \sum_{j=1}^{N} d_{(n-x)} / (N \times T) \]  

\[ F_{n-x} = \sum_{j=1}^{N} N_{(n-x)} / (N \times T) \]

When using time varying rates, (11)-(13) should not be computed for a single sub-period equal to $T$, because variation is lost; also, all SPP models have to be synchronized to a common time reference.

**V. EXAMPLES**

The proposed method is applied to a study of LCS in the transmission zone of the one-area IEEE Reliability Test System (RTS) [15] shown in Fig. 10. As pointed out by Billinton and Allan [16]: “In many cases, severity associated with a contingency event is inversely related to the frequency and the probability of its occurrence. In other words, as the number of components involved in a simultaneous outage increases, both the probability and the frequency of the contingency decrease”. By this reason, the analysis of results presented here thus focuses more on the relative change in LCS indices than in their absolute magnitudes.

**A. Example 1: Constant Rates and Diverse Repair Logistics**

Case 1: Each component has a repair team dedicated to it. Results are the same as those obtained when applying (5)-(9). Failure and repair processes of all components are modeled as HPP using RTS data.

Case 2: Three maintenance zones are considered; one for the 138 kV transmission lines, one for the power transformers and one for the 230 kV transmission lines. The repair service process in each maintenance zone follows the RTS repair data.
Case 3: Similar to Case 2, but now three operating states are considered for the power transformers, as shown in Fig 8. The nominal capacity (ONAF) state has failure rate equivalent to 80% of the value given for the RTS and a mean repair time of 768 hours (RTS data). The derated capacity (ONAN) state has failure rate equivalent to 20% the value given for the RTS and an assumed repair time of 168 hours (one week).

![Fig. 10 The Single Area IEEE RTS [15]](image)

Tables I to III shows the results for $T = 1$ year and simulations for 15000 realizations. Relative changes in results ($\Delta\%$) for case 2 are computed with reference to case 1 and for case 3 are computed with reference to case 2.

| LOCS | Case 1 | Case 2 | Case 3 | $\Delta\%$ | $\Delta\%$
<table>
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<tr>
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</thead>
<tbody>
<tr>
<td>$n=0$</td>
<td>97.910</td>
<td>97.917</td>
<td>0.03</td>
<td>97.9724</td>
<td>0.06</td>
</tr>
<tr>
<td>$n=1$</td>
<td>3.6342</td>
<td>3.6377</td>
<td>0.06</td>
<td>3.6384</td>
<td>-0.23</td>
</tr>
<tr>
<td>$n=2$</td>
<td>0.0451</td>
<td>0.0570</td>
<td>0.02</td>
<td>0.0578</td>
<td>-1.40</td>
</tr>
<tr>
<td>$n=3$</td>
<td>7.6978E-04</td>
<td>0.0011</td>
<td>0.0041</td>
<td>0.0011</td>
<td>-0.00</td>
</tr>
<tr>
<td>$n=4$</td>
<td>2.2382E-06</td>
<td>3.6916E-06</td>
<td>112.52</td>
<td>3.840E-06</td>
<td>5.17</td>
</tr>
</tbody>
</table>

These results show that:

- For the system under study, LCS of order up to 4 are expected to occur.
- When considering limited repair resources (case 2) $P_{n,x}$ of LCS or order 2, 3 and 4 have important increments.
- The effect of three-state modeling for power transformers is observed as decrements in $F_{n,x}$ and increments in $D_{n,x}$ for LCS of order 2, 3 and 4.

Required time for simulating these cases was in average 2.5 hours using normal desk computers (Intel Core 2 Processor, 2.4 and 2.66 Ghz, 2 to 3 GB of RAM).

B. Example 2: Increasing Failures Rates – Constant Repair Rates

This example considers three maintenance zones with the same repair models as in case 2 of example 1 but the failure rates are now considered increasing functions of time to represent a situation of aging and lack of maintenance.

Failure processes are modeled using a NHPP with the first intensity function shown in Fig. 7. The scale parameter $\lambda$ is defined as the permanent failure rate given as data for the RTS and a shape parameter $\beta = 1.5$. This model is called power law process or Weibull process and details about procedures to apply it to real problems are given in [17], [18].

Tables IV to VI show results for $T = 3$ year, two sub-periods per year and simulations of 12500 realizations.

Relative changes ($\Delta\%$) in the indices are shown in Figs. 11 to 13. These values are computed with reference to the index value in the first sub-period where it is non zero.

In Fig. 11 results for LCS of order zero can not be seen due to its low magnitudes. In Fig. 13 only changes in mean duration for LCS of order zero, one and two are presented; results for LCS of orders 3 and 4 are not presented because they show high oscillation.
These results show that:

- As time evolves, $n_{\alpha}P$ and $n_{\alpha}F$ for LCS of order 1 to 4 are increasing functions of time. The increments in these indexes are huge.

- For the $n_0$ operating state, $n_{\alpha}D$ is the index that better reflects the effect of increasing failure rates and constant repair resources. It is a decreasing function of time.

Required time for simulating this case was in average 6.0 hours using normal desk computers (Intel Core 2 Processor, 2.4 and 2.66 Ghz, 2 to 3 GB of RAM).

VI. THE MEANING OF LCS INDICES

It is important to clarify that $n_{\alpha}P$, $n_{\alpha}F$ and $n_{\alpha}D$ are only measurements of the uncertainty on the occurrence of LCS of a given order; hence, they only refer to the risk or the possibility of its occurrence in the long run expressed as probability, expected frequency and expected duration, respectively.

In order to measure the severity of the occurrence of a given LCS order in terms of indices such as the demand not served, the system electrical performance has to be assessed under the most severe or prevailing demand – dispatch conditions that are expected for the period $D_{n_{\alpha}}$. Obviously, this is not an easy task because each LCS order has associated many operating states and, strictly, all they have to be studied.

Thus, the LCS analysis presented here is just one step in the process of adequacy assessment a power system.
VII. CONCLUSIONS

SPP is a powerful modeling tool because it makes possible the development of a method for studying the occurrence of LCS in a power system that is more flexible than traditional methods based on Markov chains. SPP modeling can manage stationary and non-stationary failure and repair rates, common mode failures of any order, components with any number of operating states and diverse repair logistics. Additionally, a list of possible LCS it is not required as in other methods because it generates these events using a Monte Carlo simulation.

The proposed method also represents the repair process performed in each maintenance zone as it really happens, i.e. as a queuing system. This is a departure from the traditional approach of representing repairs as part of component reliability models, which implies the unrealistic assumption of a repair team per component.

Due to the huge dimension of the space of operating states of a power system, a study of LCS is always a cumbersome task no matter what method is applied. The proposed method is a very flexible modeling alternative but it requires significant computing resources.

VIII. REFERENCES


