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THIS DISSERTATION PRESENTS a method for assessing the vulnerability of a composite power system. It is based on the modeling of failures and repairs using stochastic point process theory and a procedure of the sequential Monte Carlo simulation to compute the indices of vulnerability. Stochastic point process modeling allows including constant and time-varying rates, a necessity in those scenarios considering aging and diverse maintenance strategies. It also allows representing the repair process performed in the power system as it really is: a queuing system. The sequential Monte Carlo simulation is applied because it can artificially generate all the aspects involved in the operating sequence of a power system and also because it can easily manage non-stationary probabilistic models. The indices of vulnerability are the probability of occurrence of a high-order loss of component scenario, its frequency and its duration. A high-order loss of component scenario is that one higher than  $n - 2$ . Examples using the IEEE One Area RTS show how the presence of aging and others factors that produce increasing component failure rates dramatically increase the risk of occurrence of high-order loss of component scenarios. On the other hand, the improvement in aspects such as preventive maintenance and repair performance reduces this risk. Although the main focus of this method is composite power systems, its development produced other outcomes, such as procedures for assessment of power distribution systems, protective relaying schemes and power substations.



Facultad de Ingeniería  
Departamento de Ingeniería Eléctrica y Electrónica



CARLOS J. ZAPATA

PLANNING OF INTERCONNECTED POWER SYSTEMS CONSIDERING SECURITY  
UNDER CASCADING OUTAGES AND CATASTROPHIC EVENTS

## PLANNING OF INTERCONNECTED POWER SYSTEMS CONSIDERING SECURITY UNDER CASCADING OUTAGES AND CATASTROPHIC EVENTS

A dissertation by  
**CARLOS J. ZAPATA**

Submitted to the  
School of Engineering of  
**UNIVERSIDAD DE LOS ANDES**  
in partial fulfilment for the  
requirements for the Degree of  
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**Planning of Interconnected Power Systems  
Considering Security under Cascading Outages and  
Catastrophic Events**

Esta colección reúne los mejores trabajos de grado de maestría y de doctorado de la Facultad de Ingeniería de la Universidad de los Andes. Con el ánimo de divulgar estos resultados de nuestros grupos de investigación, la Facultad los pone a disposición de la comunidad académica.

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*This work is dedicated to my parents,  
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Grisales de Zapata; to my beloved brothers  
Mr. Didier Ricardo Orduz and Mr. Harold  
Rodrigo Murillo; and to my dear teachers  
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# ABSTRACT

This work presents a method for assessing the vulnerability of a composite power system. It is based on the modeling of failures and repairs using stochastic point process theory and a procedure of the sequential Monte Carlo simulation to compute the indices of vulnerability. Stochastic point process modeling allows including constant and time-varying rates, a necessity in those scenarios considering aging and diverse maintenance strategies. It also allows representing the repair process performed in the power system as it really is: a queuing system. The sequential Monte Carlo simulation is applied because it can artificially generate all the aspects involved in the operating sequence of a power system and also because it can easily manage non-stationary probabilistic models. The indices of vulnerability are the probability of occurrence of a high-order loss of component scenario, its frequency and its duration. A high-order loss of component scenario is that one higher than  $n - 2$ . Examples using the IEEE One Area RTS show how the presence of aging and others factors that produce increasing component failure rates dramatically increase the risk of occurrence of high-order loss of component scenarios. On the other hand, the improvement in aspects such as preventive maintenance and repair performance reduces this risk. Although the main focus of this method is composite power systems, its development produced other outcomes, such as procedures for assessment of power

distribution systems, protective relaying schemes and power substations.

*Key words:* aging, interconnected power systems, Monte Carlo simulation, power distribution systems, power systems, power systems planning, power systems reliability, power systems security, protective relaying, queuing analysis, reliability, stochastic point processes, substations.

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# CHAPTER 1

## INTRODUCTION

The  $n-1$  loss of component criterion has an ubiquitous place in the study of composite power systems, for a wide variety of activities (expansion planning, adequacy assessment, security assessment, operational planning or operation), time frames (short, mid, long term or on-line) or kind of study (static or dynamic).

Surveys have long identified it as the most popular reliability criterion used by utilities.

Although other methods that allow the inclusion of high-order criteria are available,

- it still is the benchmark criterion;
- it is part of standards for transmission planning in many countries;
- it is used for supporting on-line decisions in power system operation; and
- it is also argued that it must be kept due to its popularity.

Moreover, this criterion is also embedded within probabilistic methods. Reliability assessments based on the Monte Carlo simulation and the continuous Markov process often restrict the

component outage analysis to the  $n - 1$  case, although they can handle scenarios of higher order.

Fundamentally, the preeminence of the  $n - 1$  criterion is supported by the following:

- The belief that the occurrence of more than one or two component failures over a short period is not credible. Hence, the loss of component criterion has only been extended to the  $n - 2$  case to cover common mode outages on double circuit transmission lines and voltage stability considerations.
- The fact that system operating states where one or two components are unavailable account for almost all the probability of the space of system operating states. Thus, the probabilistic “state space enumeration” method for reliability assessment of composite power systems takes advantage of this fact to speed up the computation of adequacy indices by considering only the cases and some  $n - 2$ .
- The belief that it is very expensive to plan a system which meets the requirements of power quality, service continuity and security under the loss of two or more components.

However, these items can be confronted with the following facts:

- The post-mortem analysis of some blackouts has shown that the occurrence of two or more independent component failures over a short period can occur. Hence, it is a credible situation. Also, the occurrence of two or more independent component failures can spark cascading outages leading to a blackout.
- Although the independent loss of more than two components

has a very low probability of occurrence, it can nonetheless happen.

- Many power systems currently have a significant proportion of aged components. As components are used far beyond their design life, they fail more frequently. This increases the probability of occurrence of more than one failure over a short period.
- The economic losses to consumers due to blackouts are huge. This justifies planning the power system to avoid such events.

This discussion shows that the occurrence of high-order loss of component scenarios, i.e., those higher than  $n - 2$  deserves much more attention due to its connection with cascading outages and blackouts.

This aspect can be studied using the concept of vulnerability. It is presented by an IEEE task force as [8]:

*“A vulnerable system is a system that operates with a reduced level of security that renders it vulnerable to the cumulative effects of a series of moderate disturbances.”*

*The term vulnerability is defined in the context of cascading events and therefore it is beyond the traditional concept of  $n - 1$  or  $n - 2$  security criteria.”*

Thus, in this work the security under cascading outages and catastrophic failures is studied using this concept and specifically measuring the occurrence of loss of component scenarios.

In order to develop a method for this purpose, the theory of stochastic point processes and the sequential Monte Carlo simulation were chosen for the reasons that will be explained in depth in the following chapters.

Due to the complexity of a vulnerability assessment of a composite system, the development of the method was done and is presented in this report in the following sequence:

- The concept of stochastic point process (SPP) modeling is presented in Chapter 2.
- The misconceptions about SPP and the modeling of repairable components are discussed in Chapter 3.
- The repair process in a real power system is studied in Chapter 4, in order to justify its modeling as a queuing system.
- A method for the assessment of a power distribution system is then developed and presented in Chapter 5; this is because it does not include meshed parts and does not require power flow.
- A method for the reliability assessment of protective schemes is then developed and presented in Chapter 6. It will be used in Chapter 8 to obtain the model of failure to operate of a protective system for the assessments of power systems.
- A method for the assessment of a small portion of a power system—a power substation—is presented in Chapter 7. It includes main power system apparatus and protective systems.
- Finally the method of vulnerability assessment of composite systems is presented in Chapter 9.

## CHAPTER 2

# STOCHASTIC POINT PROCESSES

### 2.1. Definition

This chapter is devoted to the theory of stochastic point processes (SPP), the modeling tool applied through this work. Most of the content of this chapter is taken from the textbook *Probabilistic Analysis and Simulation* by Zapata [17].

An SPP is a random process in which the number of events  $N$  that occur in a period of time  $\Delta t$  is counted, with the condition that one and only one event can occur at every instant.

Figure 2.1 presents a pictorial representation of the concept of an SPP, in which  $x_i$  denotes an inter-arrival interval and  $t_i$  an arrival time. If the time when the observation of the process started is taken as reference,  $\Delta t = t - 0$ , only appears in the equations that describe the process.

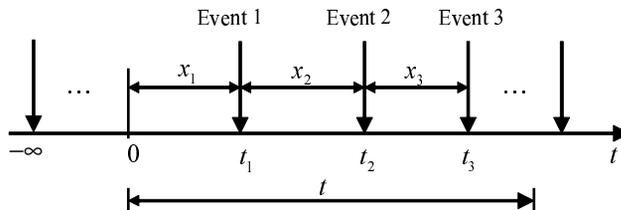


Figure 2.1. The concept of SPP

The mathematical model of an SPP is defined by the intensity function  $\lambda(t)$ :

$$\lambda(t) = \frac{dE[N(t)]}{dt}. \quad (2.1)$$

This parameter allows the calculation of:

- The expected number of events:

$$E[N(t)] = \Lambda(t) = \int_0^t \lambda(t) dt. \quad (2.2)$$

- The variance:

$$\text{Var}[N(t)] = \Lambda(t). \quad (2.3)$$

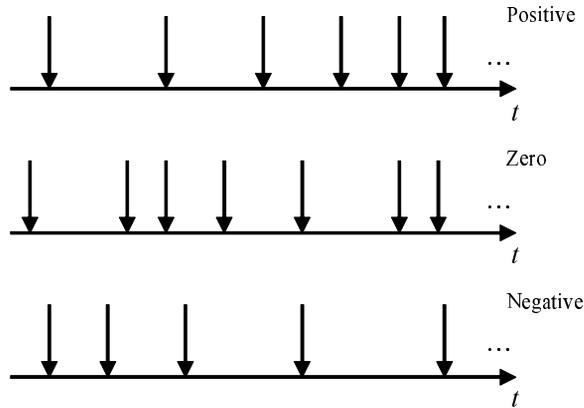
- The probability that  $k$  events occur:

$$P[N(t) = k] = \frac{1}{k!} [\Lambda(t)]^k \cdot e^{-\Lambda(t)} \quad \text{for } k = 1, 2, \dots \quad (2.4)$$

## 2.2. The Concept of Tendency

The tendency, defined as the change over time in the number of events that occur, is a very important feature of an SPP. Figure 2.2 depicts the following three kinds of tendency:

- Positive tendency: The number of events increases over time and the inter-arrival intervals decrease.  $\lambda(t)$  is an increasing function.
- Zero tendency: The number of events that occur and the inter-arrival intervals do not show a pattern of increase or decrease.  $\lambda(t)$  is constant.
- Negative tendency: The number of events decreases over time and the inter-arrival intervals increase.  $\lambda(t)$  is a decreasing function.



**Figure 2.2.** Tendency on an SPP

An SPP without tendency is stationary or time-homogeneous. Homogeneity means inter-arrival intervals are independent and identically distributed; hence, events that occur are independent. The opposite is true for an SPP with tendency.

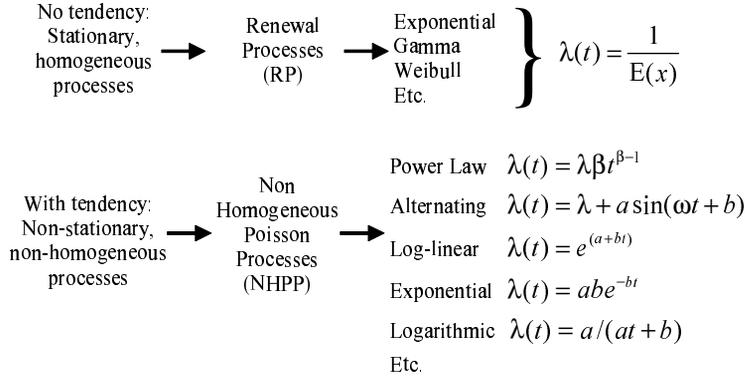
### 2.3. SPP Models

Figure 2.3 shows a basic classification of SPP models based on tendency.  $\lambda$ ,  $\beta$ ,  $a$ ,  $b$  and  $\omega$  are parameters of the models.

The name for an RP is given after the  $x$ 's distribution. The most famous RP is the exponential one, commonly called Homogeneous Poisson process (HPP).

For  $t \rightarrow \infty$ , the intensity function of every RP is a constant defined as the inverse of  $E(x)$ , the expected value of the  $x$ 's:

$$\lambda(t) = \frac{1}{E(x)}. \tag{2.5}$$



**Figure 2.3.** A basic classification of SPP models

### 2.4. Selection Procedure of an SPP Model

The procedure for fitting an SPP model to a sample data taken from a random point phenomenon is as follows:

1. By means of the Laplace test, the Mann test or graphic methods, determine if there is a tendency in the arrival or inter-arrival times.
2. If there is evidence of tendency, select an NHPP model, estimate its parameters and apply a goodness of fit test. A problem with NHPP models is that methods for parameter estimation and goodness of fit are specific to each kind of model. Furthermore, for some models no accepted method has yet been developed.
3. If there is no evidence of a tendency, apply an independence test to the inter-arrival intervals such as the scatter diagram or the correlation plot. If inter-arrival intervals are independent, fit a probability distribution using traditional methods

for parameter estimation and goodness of fit. In this case, an RP model is obtained.

## 2.5. The Power Law Process

While there are many NHPP models, the approach here is to use the Power Law Process (PLP) developed by L. Crow in 1974, because:

- it is an accepted model to represent the failure process of repairable components;
- there are methods for parameter estimation and goodness of fit;
- it can represent a process with or without tendency; and
- it can represent the HPP.

The intensity function of this process is:

$$\lambda(t) = \lambda\beta t^{\beta-1}, \quad (2.6)$$

where  $\lambda$  is the scale parameter and  $\beta$  the shape parameter, both greater than zero.

The shape parameter controls the tendency of the model in the following way:

- $\beta > 1$  for positive tendency;
- $\beta < 1$  for negative tendency; and
- $\beta = 1$  for zero tendency (in this case PLP is equal to HPP).

For a sample of  $n$  arrival times  $t_1, t_2, \dots, t_n$ , the maximum likelihood estimators of the PLP parameters are:

$$\hat{\beta} = \frac{n-2}{\sum_{i=1}^n \ln\left(\frac{t_n}{t_i}\right)}; \quad (2.7)$$

$$\hat{\lambda} = \frac{n}{t_n^{\hat{\beta}}}. \quad (2.8)$$

## 2.6. How to Generate Samples from SPP Models

### 2.6.1. Renewal Processes

1. Let  $t_0 = 0$ .
2. Generate a uniform random number  $U_i$ .
3. Get an inter-arrival interval  $x_i = F^{-1}(U_i)$  using the probability distribution function of the inter-arrival intervals.
4. The arrival time is  $t_i = t_{i-1} + x_i$ .
5. Go to step 2 until the following stopping rule is reached: A given number of events or a sample period  $T$ .

### 2.6.2. Non-Homogeneous Poisson Processes

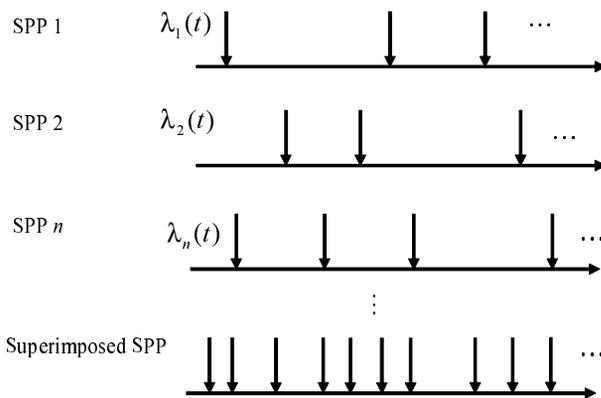
1. Generate a sequence of  $n$  arrival times from an HPP with intensity function  $\lambda = 1.0$  which covers the sample period  $T$ . These times are denoted as  $t_1', t_2', \dots, t_n'$ .
2. Find the inverse function of the mean cumulative number of events of the NHPP under study ( $\Lambda^{-1}$ ).
3. Calculate the arrival times of the NHPP as  $t_i = \Lambda^{-1}(t_i')$ .
4. Calculate the sequence of  $x$ .

The algorithm has application if the inversion of  $\Lambda$  is easy. In the case of PLP, the recursive equation is:

$$t = \left(\frac{t'}{\lambda}\right)^{1/\beta} = \Lambda^{-1}(t'). \tag{2.9}$$

### 2.7. Superposition

The operation of adding the events of several SPP for a given period  $t$  is called superposition. The resulting process is a “superimposed process.” Figure 2.4 depicts this concept.



**Figure 2.4.** The superposition of several SPP

The expected number of events during  $t$  for a superimposed SPP is the sum of the expected number of the  $NC$  SPP which compose it:

$$E[N(t)]_G = E[N(t)]_1 + E[N(t)]_2 + \cdots + E[N(t)]_{NC} = \sum_{i=1}^{NC} E[N(t)]_i. \tag{2.10}$$

The expected number of events in an SPP is obtained from its intensity function as:

$$\mathbb{E}[N(t)] = \int_0^t \lambda(t) dt. \quad (2.11)$$

Replacing(2.9) into (2.8),

$$\int_0^t \lambda_G(t) dt = \int_0^t \lambda_1(t) dt + \int_0^t \lambda_2(t) dt + \cdots + \int_0^t \lambda_{NC}(t) dt', \quad (2.12)$$

the following relationship is obtained:

$$\lambda_G(t) = \lambda_1(t) + \lambda_2(t) + \cdots + \lambda_{NC}(t). \quad (2.13)$$

The results expressed in (2.10) and (2.13) hold regardless of the kind of SPP used to compose the superimposed process.

## CHAPTER 3

# SOME MISCONCEPTIONS ABOUT SPP AND THE MODELING OF REPAIRABLE COMPONENTS

Since long ago, SPP theory has been successfully applied in many fields of knowledge such as biology, physics, queuing analysis, and engineering reliability. Statistical procedures for applying this type of modeling to real problems have been developed and several SPP models have gained wide acceptance.

On the other hand, SPP has not received as much attention in power system reliability as in other fields and only a small number of applications have been reported. This may be due to some common misconceptions about the reliability modeling of repairable components. In particular, it is often believed that SPP is identical to others widely used, such as the analyses based on the Weibull distribution.

The aim of this chapter is to bring some clarity upon SPP theory and its application in the reliability field, by discussing the origin of these misconceptions. The content of this chapter is taken from the paper “Some Misconceptions about the Modeling of Repairable Components,” by Zapata, Torres, Kirschen and Ríos [29].

### 3.1. Review of Basic Concepts

Before discussing the misconceptions, it is necessary to review some fundamental concepts about random processes.

#### 3.1.1. Definitions

The term *random process* denotes a random phenomenon that is observed in the real world. This term is reserved for a kind of modeling for random processes. The period of interest for studying a random process is denoted  $t$ . A random variable  $x$  represents the random process.

A random process is *stationary* if their statistical properties, the expectation  $E[x]$  and the variance  $\text{Var}[x]$  are constant during  $t$ . The opposite is true for a non-stationary random process.

A random process is *time homogeneous* if its probability density function  $f(x)$  does not change during  $t$ . The opposite is true for a non-homogeneous random process.

*Homogeneous* and *stationary* are interchangeable terms because: (i) If  $f(x)$  does not change during  $t$  then  $E[x]$  and  $\text{Var}[x]$  are constant during this period; and (ii) if  $E[x]$  and  $\text{Var}[x]$  are constant during  $t$ , it is necessary that  $f(x)$  does not change during this period. *Non-homogeneous* and *non-stationary* are also interchangeable terms.

A *distribution* is a mathematical model for a stationary random process in which  $t$  does not explicitly appear. A distribution is defined by means of a probability density function  $f(x)$  which does not change during  $t$ . All mathematical functions used as distributions produce  $E[x]$  and  $\text{Var}[x]$ , as this kind of model always refers to a stationary random process; hence  $E[x]$  and  $\text{Var}[x]$  are only functions of the distribution parameters which are also constant.

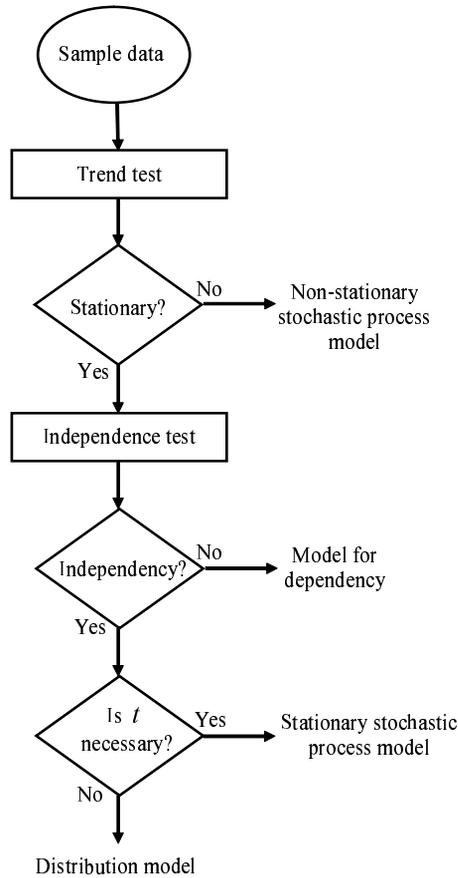
A *stochastic process* is a mathematical model for a stationary or non-stationary random process in which  $t$  appears explicitly. The random variable that represents the process can then be written  $x_t$  and  $t$  is called the process index. Thus, a stochastic process is a collection of random variables  $x_{t_1}, x_{t_2}, \dots, x_{t_N}$ , one for each value of the index  $t$ . Thus, there is a collection of probability density functions  $f_{t_1}(x), f_{t_2}(x), \dots, f_{t_N}(x)$ , one for each random variable. If for a given  $t$  the statistics of the random process are constant, it is stationary and time homogeneous because the  $f_{t_i}(x)$  do not change during this period. The opposite is true for a non-stationary, non-homogeneous random process.

### 3.1.2. How to Select a Model for a Random Process

Figure 3.1 shows the basic procedure for selecting a model that is a proper representation of a random process. Omitting any of the three steps of this procedure can lead to an unsuitable model. A sample  $x_1, x_2, \dots, x_n$  is the input data for this procedure.

The first step is to determine whether the random process is stationary or non-stationary. Several statistical methods are available for this. However, only trend tests are discussed here, because only sequences of times to failure (*t<sub>t</sub>f*) and times to repair (*t<sub>t</sub>r*) are considered in this chapter.

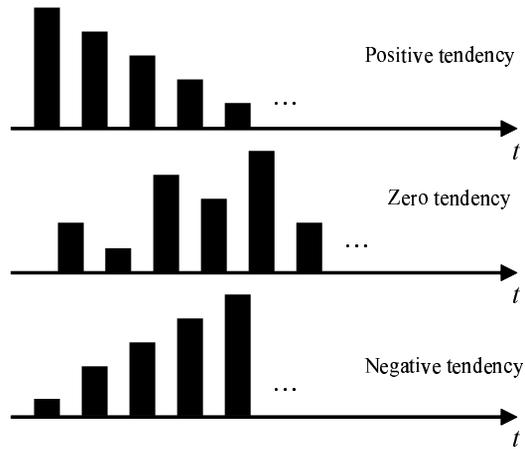
Figure 3.2 shows a simple trend test where the bar graph shows the chronologically ordered, inter-arrival time magnitudes. If this graph shows a pattern of increasing or decreasing inter-arrival time magnitudes, then the random process is deemed to have a tendency, or else it is non-stationary. If this test does not show that the random process has a tendency, it is deemed to be stationary. The basic condition to guarantee the validity of a trend test is to keep the chronological order in which the inter-arrival times



**Figure 3.1.** Procedure to select a model for a random process

occurred.

If the sample data for the random process shows that it is non-stationary, then a non-stationary stochastic process model has to be selected. This can be done by applying the procedures for parameter estimation and a goodness of fit test, which are particular for each model in this class and should not be confused with the ones used for distributions. Two important families of non-stationary stochastic processes are the non-homogeneous Markov



**Figure 3.2.** Bar graphs of inter-arrival times magnitudes for trend test

chains and the non-homogeneous Poisson processes.

If the sample data for the random process under study shows it is stationary, it is necessary to apply a test for independency, such as the scatter diagram or the correlation plot. Two cases arise here:

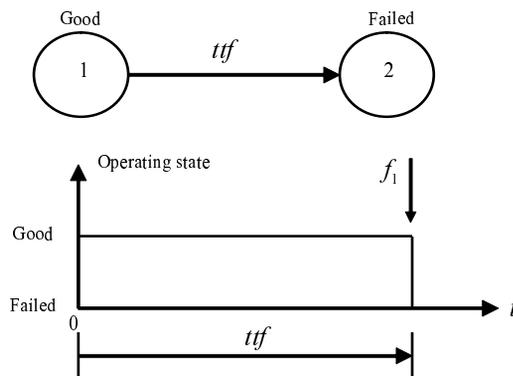
1. If the sample data is not independent, a model for dependent events has to be selected. An example of these kinds of models is the branching point process or time series.
2. If the sample data is independent, a distribution must be selected if  $t$  is not necessary to explain the random process. If that is not the case, a stationary stochastic process must be selected. In both cases it is necessary to apply the procedures for parameter estimation and a goodness of fit test to select the distribution or the stationary stochastic process model that can represent the random process under study.

The importance of performing trend and independency tests is discussed by Ascher and Hansen [2] who point out that:

1. It is incorrect to fit a sample of inter-arrival times to a distribution model without performing first a trend test to check that the random process from which the sample was taken is stationary. Goodness of fit tests sorts out sample values by magnitude, hence losing the chronological order in which they occurred.
2. It is incorrect to fit a sample of inter-arrival times to a distribution model without performing first an independency test because the goodness of fit tests—such as chi square and Kolmogorov-Smirnov—were developed assuming sample independency. This also applies to the maximum likelihood method for parameter estimation.

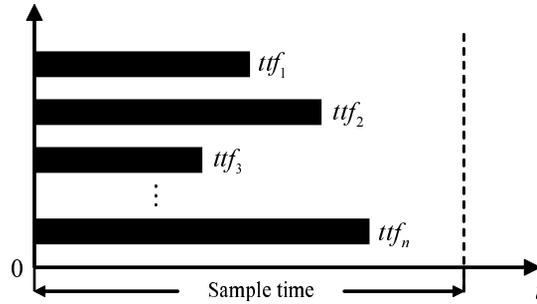
### 3.2. Reliability Analysis of Non-Repairable Components

A non-repairable component is one that dies when the first failure  $f_1$  occurs. The classical model for this kind of component is shown in Figure 3.3. It only considers two operating states and  $tff$  is used to represent the failure process.



**Figure 3.3.** Operating states of a non-repairable component

Because a non-repairable component can fail only once, a sample  $ttf_1, ttf_2, \dots, ttf_n$  obtained from a group of identical components that have failed is necessary to build its reliability model. Figure 3.4 shows such a sample. These values are not ordered in a chronological sequence and each has no connection with the other sample values. Furthermore, the instant when the observation of the operating time was taken does not matter.



**Figure 3.4.** Sample of  $ttf$  of a group of identical non-repairable components

The  $ttf$  sample is fitted to a distribution  $f_{ttf}(t)$  that is called *life model*.  $F_{ttf}(t)$  gives the probability of failure and its complement  $R_{ttf}(t) = 1 - F_{ttf}(t)$  is the reliability.

One important aspect to study for non-repairable components is the risk that a component that has not failed until a given time fails after it. This is a conditional probability that leads to the famous equation for  $\lambda(t)$  called “failure rate” or “hazard rate”:

$$\lambda(t) = \lim_{\Delta t \rightarrow 0} \frac{R(t) - R(t - \Delta t)}{\Delta t \cdot R(t)} = \frac{f_{ttf}(t)}{[1 - F_{ttf}(t)]}. \quad (3.1)$$

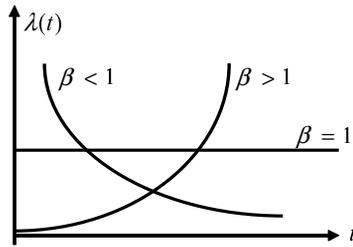
Depending on the kind of distribution used for the life model or the values of its parameters,  $\lambda(t)$  can be constant or a function of time. Only for the exponential distribution  $\lambda(t)$  it is a constant;

for a Gaussian distribution it is an increasing function of time, etc. For a Weibull distribution with scale parameter  $\lambda$  and shape parameter  $\beta$ ,  $\lambda(t)$  is defined by (2). As shown in Figure 3.5 the form of  $\lambda(t)$  depends on the value of  $\beta$ .

$$\lambda(t) = \lambda\beta t^{\beta-1} \quad (3.2)$$

As can be seen, (3.2) is the same as (2.6)—the intensity function of a Power Law process.

Equation (3.2) has a ubiquitous place in reliability. Unfortunately, as it will be discussed later, this has led some authors to forget its real meaning and origin.



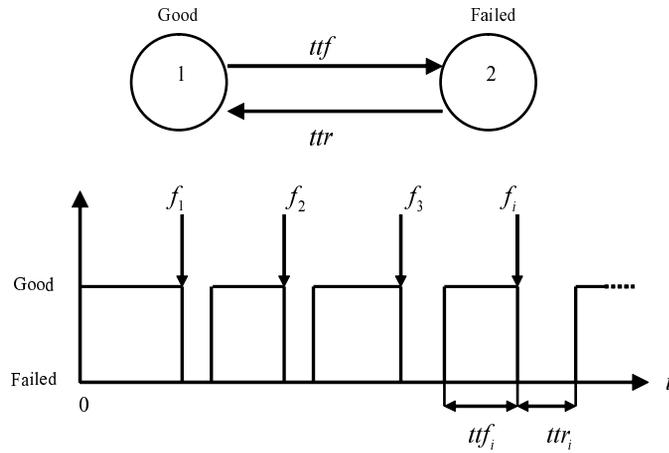
**Figure 3.5.** Failure rate for a non-repairable component with Weibull life model

### 3.3. Reliability Analysis of Repairable Components

A repairable component is one that can withstand a sequence of failures  $f_1, f_2, \dots, f_n$ . Its simplest representation in terms of reliability is the two-state diagram shown in Figure 3.6.

The failure rate of a repairable component is the rate of change of the expected number of failures  $N$  in a given period  $t$ :

$$\lambda(t) = \frac{dE[N(t)]}{dt}. \quad (3.3)$$



**Figure 3.6.** Two-state diagram and operating sequence of a repairable component

The independent processes of failures and repairs can be illustrated by the operating sequence shown under the two-state diagram in Figure 3.6. Unlike the case of a non-repairable component, in this case the sample values  $ttf_1, ttf_2, \dots, ttf_n$  and  $ttr_1, ttr_2, \dots, ttr_n$  must be chronologically ordered sequences to keep the tendency of the failure and repair processes.

The two main families of models that have been applied to the reliability analysis of repairable components are discussed next.

### 3.4. Markov Chain Models

The term *Markov chain* refers here to a family of models which couples the processes of failures and repairs in a two-state diagram representation such as the one shown in Figure 3.6. This definition is adopted because there is no general agreement about the names for the different extensions for the basic continuous-time, exponential Markov chain model.

### 3.4.1. Homogeneous Exponential Markov Chain

If the samples of  $t\bar{t}f$  and  $t\bar{t}r$  show no tendency, are independent and meet a goodness of fit test for exponential distributions with parameters  $\lambda = 1/\overline{t\bar{t}f}$  and  $\mu = 1/\overline{t\bar{t}r}$ , respectively, then the coupled process of failures and repairs is described by:

$$\begin{pmatrix} \frac{dP_1(t)}{dt} \\ \frac{dP_2(t)}{dt} \end{pmatrix} = \begin{pmatrix} -\lambda & \mu \\ \lambda & -\mu \end{pmatrix} \begin{pmatrix} P_1(t) \\ P_2(t) \end{pmatrix}. \quad (3.4)$$

$P_1(t)$  and  $P_2(t)$  are the probabilities of finding the component in states 1 (good) and 2 (failed), respectively.  $\lambda$  and  $\mu$  are called “failure rate” and “repair rate,” respectively, or more generally “transition rates.” Overline symbols denote a statistical mean. The most appealing characteristic of this model is that it has an analytical solution.

This model is memoryless or Markovian, i.e., the transition to another state depends only on the current state and thus the trajectory before reaching the present state does not matter. This model is commonly called *homogeneous Markov process* or *homogeneous Markov chain*.

### 3.4.2. General Homogeneous Markov Chain

In this case, samples of  $t\bar{t}f$  and  $t\bar{t}r$  show no tendency, are independent, and one or both of them meet the goodness of fit test with a non-exponential distribution. When both distributions are not exponential, this model is called *non-Markovian process* and for the case where one is exponential but the other not it is called *semi-Markov process*. We adopt the name *general homogeneous Markov chain* because “general” indicates that any kind of distributions can be used and “homogeneous” specifies that these

distributions do not change over time. This model does not have the memoryless property, i.e., it is non-Markovian and cannot be solved using (3.4). Solution methods include the Monte Carlo simulation, the device of stages and the technique of adding variables.

This model is very important because it is unusual for both the failure and repair distributions to be exponential. While the failure process for non-aged components generally fits an exponential distribution, repair times are generally lognormally distributed.

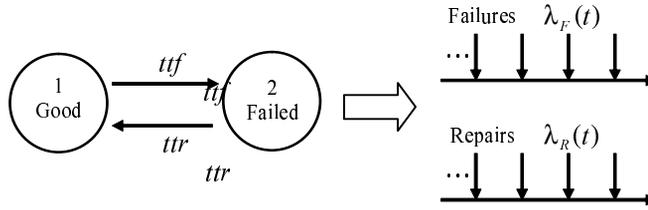
### 3.4.3. Non-Homogeneous Markov Chain

In this case,  $\lambda$  and  $\mu$  in (3.4) are not constant but functions of time. The failure and repair processes are thus not homogeneous because, as time passes, the expected number of failures and the expected number of repairs are not constant. Therefore, the failure and repair processes cannot be represented by means of distributions. This model also does not have the memoryless property, i.e., it is non-Markovian. Popular solutions to this process are numerical methods of differential equations and the sequential Monte Carlo simulation. However, it has problems for adjusting the operating times, as well as of tractability of some types of time-varying rates [6].

### 3.4.4. SPP Models

As shown in Figure 3.7, this kind of modeling decouples the processes of failures and repairs of the component. Failures and repairs are represented by sequences of events that arrive independently.

In many applications the repair process is neglected because the repair times are much shorter than the typical interval of time-separating failures. For example, the repair time may be of the



**Figure 3.7.** In SPP modeling the process of failures and repairs are uncoupled

order of hours, compared to times to failure in the order of years.

### 3.5. The Misconceptions

#### 3.5.1. The Meaning of the Term “Failure Rate”

The first problem that arises is that the failure rate given by (3.1) is confused with the one in (3.3) when an SPP is used to model the failure process of a repairable component. The two concepts are different:

1. The failure rate (3.1) refers to failures that affect a population of identical non-repairable components and kill them. For a single non-repairable component, it can neither be calculated nor measured.
2. The failure rate (3.3) refers to failures that affect a single repairable component if the sample was taken from a particular component. Also, it can refer to failures that affect a population of identical or non-identical repairable components if component failure data were pooled.

In order to distinguish the two concepts, Ascher and Feingold [1] proposed the acronym ROCOF (rate of occurrence of failures) to denote (3.3). While this lexical distinction is useful, it

is essential to understand what definition applies for repairable and non-repairable components; it is incorrect to use the definition (3.1) for repairable components, or the definition (3.3) for non-repairable ones. However, in many papers, (3.1) is presented as the failure rate of components that are repairable, such as power transformers and generators. In [15] Thompson discusses the uses and abuses on the application of (3.1).

### 3.5.2. The Use of a Life Model for a Repairable Component

The life model of a non-repairable component  $f_{ttf}(t)$  refers to the arrival of one and only one failure that kills it. Thus, it is incorrect to apply this concept to a repairable component, as it can withstand several failures. But what happens if an analyst takes a sample of  $ttf$  from a repairable component and, after applying required tests, shows that a given distribution is a valid representation of this failure process and calls it the component's life model with failure rate defined by (1)? Although the procedure is correct, the way the analyst conceives the model is flawed:

1. As explained before, the failure rate (3.1) does not apply.
2. The distribution represents the inter-arrival times of failures. It can be used to calculate the probability that  $ttf$  is less or equal than a given value, for generating a sequence of time to failures or for defining an RP failure model with failure rate given by (3.3).
3. The distribution is not a life model because it does not define the death of the repairable component. Such an event is defined mainly by economic considerations: A failed component is deemed to have died and is thus replaced if its repair

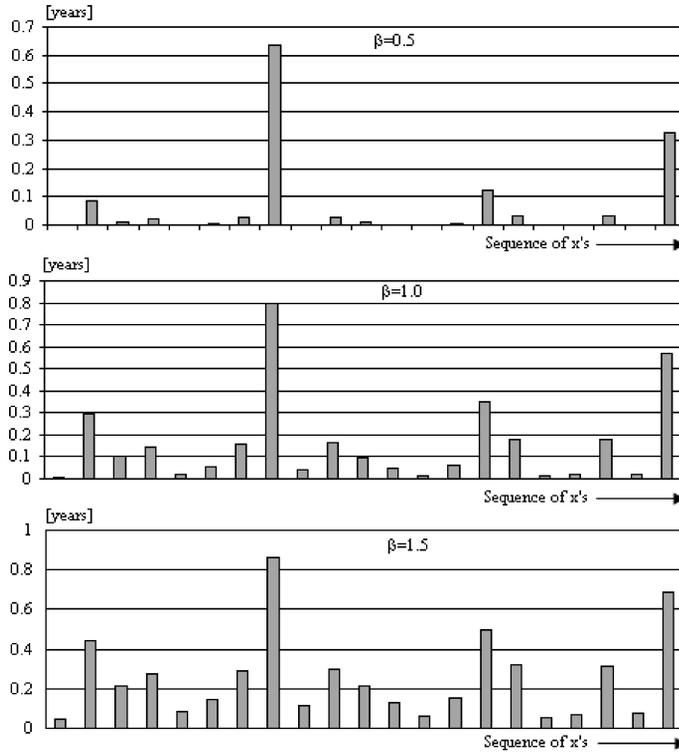
cost is equal or higher than its replacement cost, or if the expected cost of its unavailability during a planning period is higher than its replacement cost.

### 3.5.3. A Distribution Can Represent a Non-Stationary Random Process

This is the most misleading idea in reliability! A distribution can only be used to model stationary random processes. All mathematical functions used as distributions produce constant statistics. This fact can be easily proven using a bar diagram of a sequence of values generated from any distribution. Figure 3.8 shows this for a realization of a Weibull distribution with  $\lambda = 5$  [years] and different values of  $\beta$ . As it can be seen there is no tendency in any case.

Similarly, RP are always stationary because they are defined on the basis of the distribution of inter-arrival times. Thus, Thompson [15] points out that an RP cannot model component aging and discusses this misconception.

This misconception originates from (3.1); as it can produce increasing or decreasing failure rates depending on the kind of distribution or in accordance with the value of its shape parameter, it is believed (or, more precisely, misbelieved) that this is a natural property of some distributions. Thus, in some papers a time-varying failure rate is defined for a repairable component and, without a theoretical support, the *t<sub>t</sub>f* are generated using an exponential or Weibull distribution.



**Figure 3.8.** Bar graphs of the values generated from a Weibull distribution

#### 3.5.4. Equation (3.2) Generates a Random Process Whose Model is the Weibull Distribution

This misconception is a consequence of the previous one. The truth is that, if (3.2) is used as an intensity function for an SPP or as a transition rate for a Markov chain, then an HPP is obtained when  $\beta = 1$ , and a non-stationary one when  $\beta \neq 1$ . This can be proven using the algorithm given in section 2.6.2. More importantly, this is valid for any random process and not only for those which pertain to failures. The relationship between the Weibull distribution and (3.2) is restricted only to the case where the reliability of a non-repairable component is studied.

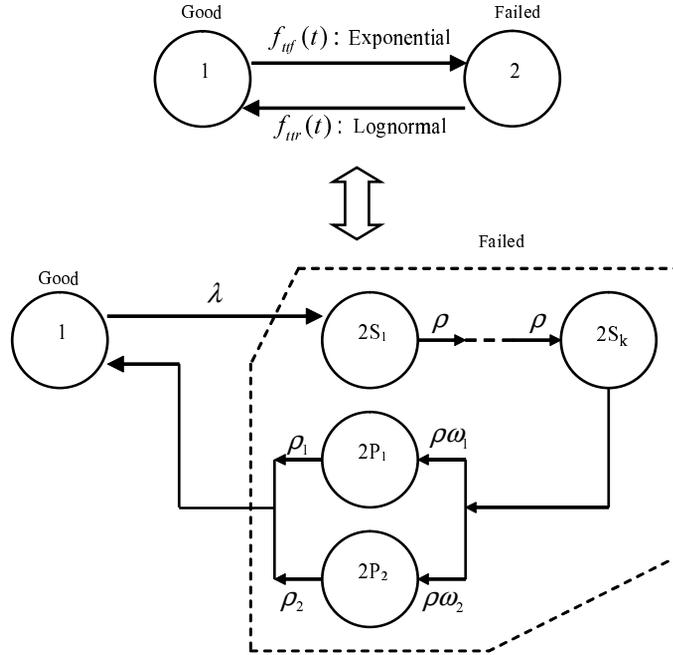
This misconception originates from the fact that the concept expressed by (3.2) has been applied extensively in the reliability field, forgetting in many cases its origin and meaning. For example:

1. Many books and papers show it as a natural property of the Weibull distribution. Results obtained by means of (3.2) are only valid when referring to the reliability of a non-repairable component, a particular result of an application where the Weibull distribution is applied.
2. Many papers define the failure rate for a repairable component using (3.2) and assert that it belongs to the Weibull distribution, although they are applying a proper method for a non-stationary analysis. That is, the analysis is correct but they are bringing a concept that does not apply.

### 3.5.5. A General Homogeneous Markov Chain Can Represent a Non-Stationary Process

This misconception is also a consequence of the third misconception. It is not true because a distribution always refers to a stationary process. The bar diagram shown in Figure 3.8 proves this for a Weibull distribution. In addition, let us consider now the method called the *device of stages*, viz., for some pairs of distributions (exponential-lognormal, exponential-Weibull, etc.), it transforms the two-state general homogeneous Markov chain in an exponential one that has more than two states. Figure 3.9 shows an example: The exponential-lognormal chain is transformed into an exponential one where state 2 is replaced by  $k$  stages in series ( $2S_1$  to  $2S_k$ ) and two stages in parallel ( $SP_1$  and  $2P_2$ ).

Transition rates  $\rho$ ,  $\rho\omega_1$ ,  $\rho\omega_2$ ,  $\rho_1$ , and  $\rho_2$  are constants obtained from the four first moments of the lognormal distribution. If the



**Figure 3.9.** The device of stages for solving a given homogeneous Markov chain

equivalent exponential Markov chain obtained using the device of stages, which is stationary, solves the two-state general homogeneous Markov chain, how can the latter be non-stationary? However, some papers apply the device of stages and say that for the case Weibull-lognormal it represents a non-stationary process!

This misconception originates from incorrectly believing that the transition rates of a general homogeneous Markov chain are defined by means of (3.1). This is wrong because there is no connection between the concepts of transition rate of a Markov chain and hazard rate of a non-repairable component. Concept (3.1) cannot be extended to failures of a repairable component nor to other events such as repairs.

### 3.5.6. The PLP is the Same Thing as a Weibull Distribution

The arguments presented in section 3.5.4 show that this is false. The PLP has no connection with the Weibull distribution. The origin of this misconception is the fact that the PLP intensity function is the mathematical function (3.2). However, when applying (3.2) the context of application should be remembered:

1. For a non-repairable component, it refers to a sequence of failures that affect a population of identical non-repairable components, not to the process of failure arrivals to a single non-repairable component nor to the arrival of other, non-failure events.
2. For a repairable component, it refers to a sequence of events that arrive. It is not confined to the case of failures. And in the case of failures, it can represent the process of failure arrival to a repairable component or to a population of repairable components.

### 3.5.7. The PLP is the Same Thing as a Weibull RP

The arguments presented in section 3.5.5 can be used to show that this is a misconception. In a PLP an exponential stationary process is obtained when  $\beta = 1$  and a non-stationary one when  $\beta \neq 1$ . When  $\beta = 1$  it generates a HPP, not a Weibull RP. This misconception has the same origin that the one discussed in section 3.5.6.

Another factor that reinforces this misconception is that PLP has received other names with the word Weibull such as Weibull process, Weibull-Poisson process, Rasch-Weibull process [9].

### 3.5.8. The Only Model for a Stationary Failure Process is the HPP

This is probably the most common of all misconceptions, but it is not as misleading as the one discussed in 3.5.4.

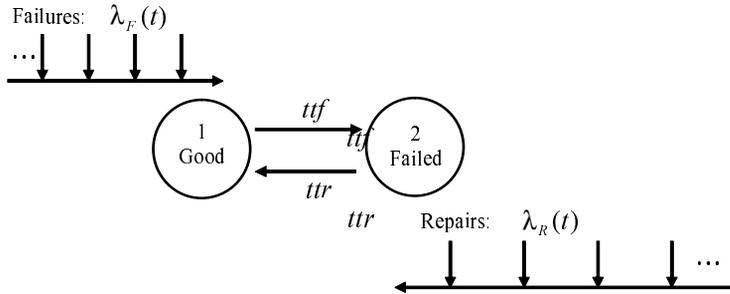
This statement is only valid when a sample of  $tff$  taken from repairable component shows no tendency, is independent and complies with the goodness of fit test for an exponential distribution. But what happens if the sample shows no tendency, is independent, but does not comply with a goodness of fit test for the exponential distribution? In this case, it is incorrect to assume an exponential distribution; the failure process of the repairable component has to be represented by means of the RP of a distribution that satisfies a goodness of fit test.

This misconception originates again from the concept of failure rate for a non-repairable component (3.1); it produces a constant failure rate only for the case of an exponential distribution. Thus, “constant failure rate = HPP model” has been applied as a rule of thumb for any type of components, forgetting that this result was obtained only for non-repairable ones. For the case of a repairable component with stationary failure process, all RP are possible failure models.

## 3.6. Relationship Between SPP and Markov Chains

A two-state Markov chain is generated by two SPP processes, as shown in Figure 3.10. Every time a failure arrives to the component, it is sent from the good state to the failed one; and every time a repair is performed, the component comes back to the good state. The sources of this motion are the SPP.

Intensity functions  $\lambda_F(t)$  and  $\lambda_R(t)$  in the SPP models are equal to transition rates  $\lambda_{12}(t)$  and  $\lambda_{21}(t)$  in the Markov chain,



**Figure 3.10.** Relationship between a two-state Markov chain and SPP

respectively, regardless of whether the models are defined using distributions or non-stationary stochastic processes.

One could therefore argue that, since both types of models are equivalent, there is no reason to use an SPP when Markov chains are a more popular method. While this would be true at the component level, analysts usually deal with systems of repairable components. When dealing with large repairable systems, the repair process should not be included in the component level because:

1. It is equivalent to assume repair resources are unlimited because every time the component fails a crew is available to repair it, or in other words, there is a repair team dedicated to each component. Hence, an implicit assumption is made that repair times depend only on the particular actions taken to fix each type of component.
2. For maintenance purposes, a power system is usually split into several zones or service territories, and repair teams are assigned to each area. The repair process performed in each service territory is really a queuing system.

SPP modeling thus makes it possible to represent the repair process performed in each area of a large repairable system as it

really happens. This is something that Markov chain modeling is unable to do.

### 3.7. Conclusions

There are several common misconceptions about the modeling of repairable components for reliability studies. In particular, it is often assumed that SPP are identical to other methods currently in widespread use, for example, the popular analyses based on the Weibull distribution.

All these misconceptions originate in the incorrect practice of analyzing the reliability of repairable components using concepts that were developed only for non-repairable ones and, specifically, in the misleading idea that a stationary random process model can represent a non-stationary random process.

Reliability engineers must consider carefully the concepts of homogeneity and stationarity of random processes, the procedure for selecting a type of model for a random process, and the differences between the main types of models that are available.



## CHAPTER 4

# THE REPAIR PROCESS IN A POWER SYSTEM

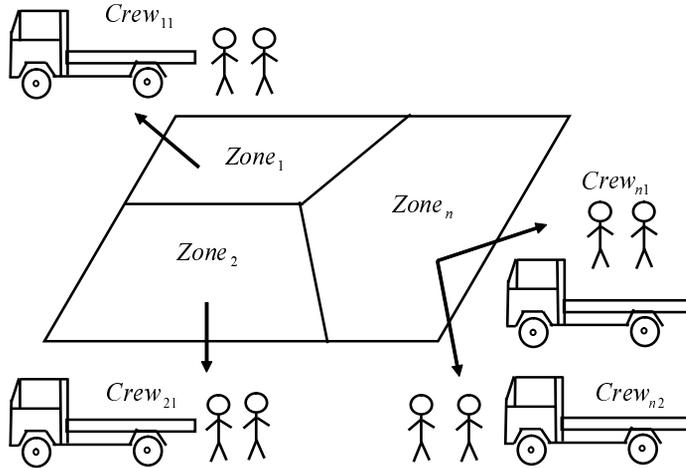
The aim of this chapter is to show why the repair process performed in a power system should not be modeled as part of the component reliability models but independently using queuing theory concepts. This also justifies the use of SPP as a modeling method for reliability assessments. In order to deepen this subject, data of the repair process performed in three Colombian distribution systems was used to apply the proposed approach of modeling.

The content of this chapter is taken from the paper “Modeling the Repair Process of a Power Distribution System,” by Zapata, Silva, González, Burbano and Hernández [26].

### 4.1. Introduction

For maintenance purposes the power distribution system is split into several zones or service territories, each one assigned to a repair team, as shown in Figure 4.1.

The resources for repairs are the personnel, trucks, tools, spares, etc. available for this work. The way these resources are organized,



**Figure 4.1.** Zones for maintenance in a power distribution system

for example, the number of crews for each zone, is the logistics. The repair resources generate the repair process.

The repair process is the sequence of repairs performed by crews in accordance with the repair orders sent by the control center, which either automatically detects component failures or receives customer calls regarding service interruptions. Thus, the repair process in each service territory is a queuing system. The input to this system is the sequence of component failures which produce service interruptions that have to be repaired by crews. The output of this system is the sequence of service restorations performed by crews.

The performance of the repair process is dependent on the quality and quantity of repair resources and the logistics. These resources are limited and have to be carefully matched to follow the pace of component failures in order to obtain acceptable outage times.

Another important thing to point out is that traditional reliability assessment of power systems has usually not considered

low-voltage components. However, repair teams also have to repair failures on these components and this increases the demand on the repair process. Moreover, as shown in Table 4.1, in some power systems, low-voltage components are the ones that fail most frequently [23]. Thus, it is very important to include these components in the reliability assessment.

Voltage level	Component	Total	
Medium	13.2 kV feeders	6.7%	22%
	33 kV feeders	0.2%	
	Distribution transformers	9.6%	
	Cut-outs and disconnectors	4.3%	
	Surge arrester and substation equipment	1.5%	
Low	Secondary distribution	30.9%	78%
	Customer drops	23.4%	
	Customer energy meters	9.4%	
	Customer installation	11.8%	
	Public lighting components	2.3%	
Total		100%	

**Table 4.1.** Components which caused service interruptions in Pereira, Colombia (2000–2002) [23]

Traditional methods for studying the repair process of a power distribution system do not model it as it really is. Thus, the subject of this chapter is the modeling of repair processes, using concepts of queuing theory and stochastic point processes.

## 4.2. Traditional Methods for Studying the Repair Process

The repair process performed in a power distribution system has been traditionally studied in the following ways:

#### **4.2.1. By Means of Statistical Analysis of Outage Times**

These kinds of studies take operating data of the power distribution system and analyze the statistics of outage times by feeders, substations and geographical zones to give guidelines about which zones of the system need improvement on the repair process performance.

Although these kinds of studies can include the modeling of the outages times using probability distributions or stochastic processes, they do not include an explicit modeling of the repair process.

As a service territory can include parts of several feeders, these kinds of studies have to be extended to each service territory because, in this case, a global analysis can be misleading.

#### **4.2.2. As Part of the Component Reliability Models**

This approach is extensively applied in power distribution reliability assessments no matter the methodology (cut-sets, analytical simulation, the Markov process or the Monte Carlo simulation): the repair process is included as part of component reliability models, by means of the probability distribution of times to repair.

This approach has the following disadvantages:

- No matter the probability distribution used, it is assumed repair resources are unlimited because every time a component fails a crew is available to repair it. Hence, the repair time only depends on the particular actions taken to fix each kind of component.
- As the repair process is represented by means of a probability distribution, it is assumed it is a stationary process, i.e., the performance of repair teams is not affected by in-

ternal or external factors. However, in real life, the crew performance is affected by external factors—weather, traffic, etc.—and also by internal factors—available tools, available skills, workload, etc.

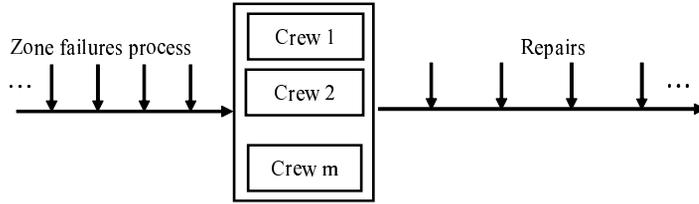
- The tendency of the repair process performed in the power distribution system is lost because the times to repair are classified by component type and thus the chronological sequence in which they occur is lost.
- Most methods apply the  $n - 1$  loss of component criteria. That is not a true assumption because a failure can occur independently, if other failures which occurred before have been repaired or not.
- Reliability assessments of power distribution systems only include high-voltage and medium voltage components. In real life, however, repair teams also have to repair the low-voltage components, a fact that causes an important demand on repair resources. Moreover, reliability surveys shows that in some power distribution systems the low-voltage components are the ones that fail more frequently.

### 4.3. Modeling of the Repair Process

The repair process of each zone (service territory) of a power distribution system is modeled as the queuing system shown in Figure 4.2.

For this queuing system the following is defined:

- **Clients:** Failures which produce service interruptions and have to be repaired by crews.



**Figure 4.2.** Queuing model of the repair process in a service territory of a power distribution system

- **Resources:** The number of crews in the zone. A crew corresponds to a server in queuing theory terminology.
- **Capacity:** Infinite, because all the failures considered here have to be repaired.
- **Queuing discipline:** First come, first served (FCFS).
- **Input process:** The zone failure process. It is the superposition of the failure processes of the components located in the zone. Only failures which produce service interruptions and have to be repaired by a crew are considered. This process has a failure intensity  $\lambda_F(t)$ .
- **Service process:** The SPP that represents the equivalent capacity of all crews assigned to the zone in the form of a repair intensity  $\lambda_R(t)$ .
- **Output process:** The SPP of the repairs performed by crews. These repairs are related to service restorations. The output process is the result of the interaction between the input and the service processes.

Using Kendall's notation, this queuing system is described as follows:

$$G / G / m / \infty / FCFS .$$

The first and second  $G$  indicate that both the input and service processes are general SPP (RP or NHPP).  $m$ ,  $\infty$ , and  $FCFS$  indicate, respectively, the number of crews, the system capacity and the queuing discipline.

The traffic intensity index  $a(t)$  is defined as:

$$a(t) = \frac{\lambda_F(t)}{\lambda_R(t)}. \quad (4.1)$$

Although  $a(t)$  is dimensionless, it is measured in Erlangs. A traffic intensity of 1.0 Erlang means one failure uses or occupies the repair resources 100% of the time. Traffic intensity higher than 1.0 means the failures arrive faster than repairs can be performed. Thus,  $a(t)$  have to be less or equal to 1.0 in order to have a stable queuing system.

#### 4.4. Assessment of the Repair Process Performance

##### 4.4.1. Obtaining the Zone Failure Process

From operating records, obtain a sample of arrival times of those component failures which caused service interruptions and were repaired by crews. It is recommended the sample covers at least one year of system operation.

It is important to remember that:

- Not all service interruptions are solved by crews; some of them are solved by means of a reconnection performed by a circuit breaker or recloser.
- Low-voltage components also cause service interruptions which in most of the cases have to be repaired by crews.

Apply to the failure arrival-times sample the procedure for selecting an SPP model. In accordance with the tendency on the resulting zone failure process, the following can be concluded:

- **Zero tendency:** The population of components located in the zone is in its useful life. That is, their reliability is neither improving nor deteriorating.
- **Positive tendency:** The population of components located in the zone shows aging.
- **Negative tendency:** The population of components located in the zone shows reliability improvement.

This kind of modeling implies repairs are minimal, i.e., they only return the components to the operating state without improving or deteriorating their reliability condition.

#### 4.4.2. Obtaining the Zone Service Process

For each failure that caused a service interruption and was repaired by means of a crew action, obtain the time to repair ( $ttr$ ).

Time to repair includes: transportation time to the place where customers are without service, time to find the failed component, time to fix the failed component and reconnection time.

A  $ttr$  does not include the waiting time ( $tw$ ) the period during which the crew receives the repair order and is free to go to repair the failure. The waiting time is a result of the congestion on the repair process, i.e., the fact that when a crew receives a repair order, it can be busy repairing a failure that occurred before.

Apply to the sample of times to repair the procedure for selecting an SPP model. In accordance with the tendency on the resulting service process the following can be concluded:

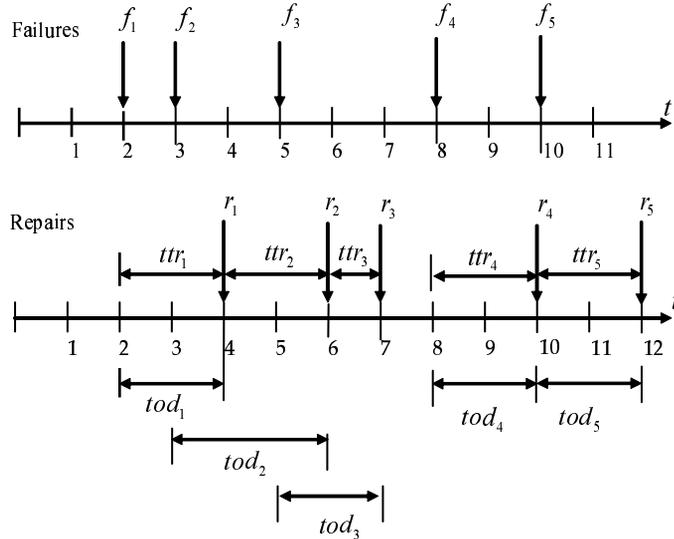
- **Zero tendency:** The crew performance is neither increasing nor decreasing.
- **Positive tendency:** The crew performance is increasing because, as time passes, repairs takes less time to be performed.

- **Negative tendency:** The crew performance is decreasing because, as time passes, repair takes more time to be performed.

**4.4.3. Assessing the Repair Process Performance**

The repair process is observed artificially for a period  $T$  of one or more years by means of a sequential Monte Carlo simulation procedure. A simulation consists of  $N$  iterations or artificial observations of the repair process performance during  $T$ .

In each iteration, the sequence of component failures and repairs is generated using the input and service processes. Figure 4.3 shows the interaction between the failure and service processes for a zone with one crew or one equivalent crew.



**Figure 4.3.** Calculation of outage durations

Every time a failure  $f_i$  arrives, it is assigned to a crew that performs a repair  $r_i$  in a time  $ttr_i$ . The arrival times of  $f_i$  and  $r_i$  are  $tf_i$  and  $tr_i$ , respectively.

Each iteration produces a sample of number of failures ( $nf$ ), times to repair ( $ttr$ ), outage duration ( $tod$ ) and waiting times ( $tw$ ).

Two stopping rules may be used for the simulation: a fixed number of iterations, or the coefficient of variation of a load point index.

#### 4.4.4. Iteration Procedure

1. Generate the input process for a period  $T$  using the zone failure SPP model.
2. Generate the service process. That is, for each failure  $f_i$  generate a  $ttr_i$  using the zone service SPP model.
3. Compute the mean traffic intensity  $ma(t)$ .
4. The arrival time of the first repair is:

$$tr_1 = tf_1 + ttr_1 . \quad (4.2)$$

5. The arrival time of the next repair is determined in the following way:

- **Congestion:** If all crews are busy when failure arrives, this failure has to wait until some crew finishes a repair  $j$  and fixes it:

$$tr_i = tr_j + ttr_i . \quad (4.3)$$

- **No congestion:** If a crew is free when failure arrives, the repair for this failure starts immediately.

$$tr_i = tf_i + ttr_i . \quad (4.4)$$

6. Calculate the outage duration:

$$tod_i = tr_i - tf_i. \quad (4.5)$$

7. Calculate the repair waiting time:

$$tw_i = tod_i - ttr_i. \quad (4.6)$$

8. For  $T$  or its sub-periods (month, semester, etc.) compute the mean waiting time ( $mtw$ ), the mean outage duration ( $mtod$ ) and the congestion ( $C$ ) defined as:

$$C = \frac{mtw}{mtod} \cdot 100\%. \quad (4.7)$$

#### 4.5. Examples

Traditional queuing analyses assume the input and service processes are Markovian (both HPP) or semi-Markovian (one HPP and the other an RP). However, for the repair process of a power distribution systems it is not known which SPP models can represent this processes.

Thus, data of three Colombian power distribution systems was gathered in order to know these models and to apply the proposed methodology. Table 4.2 shows general description of the studied systems.

For each system an assessment of the repair process performance was carried out for  $T = 1.0$  year with simulations of 150 iterations. Tables 4.3 to 4.8 show the results. Confidence level of input and service process models is 95%.

These results show the following:

	System		
	Pereira	Pasto	Casanare
Region	Municipality of Pereira	Municipality of San Juan de Pasto	Department of Casanare
Utility	Empresa de Energía de Pereira S. A.	Centrales Eléctricas de Nariño S. A.	Empresa de Energía del Casanare S. A.
Area [km <sup>2</sup> ]	702	1181	44640
Urban population	371239	312377	200952
Rural population	72315	70341	94401
Service territories	3	1	3
Crews per zone	3	2	2

Notes:

1. In Colombia a department is a group of municipalities
2. The capital of Department of Casanare is Yopal city (Inhabitants: 90218 urban and 16604 rural)
3. Population is given in inhabitants
4. Source for population data: Colombian census of year 2005 ([www.dane.gov.co](http://www.dane.gov.co))
5. Casanare Department is mainly grassland plains with very difficult transport conditions during rainy season which last at least 6 months.

Table 4.2. General data of studied systems

Zone	Input process [Failures/hour]	Service Process [Repairs/hour]
1	PLP $\lambda_p = 0.1471 \quad \beta_p = 1.0539$	PLP $\lambda_r = 0.3765 \quad \beta_r = 1.0291$
2	PLP $\lambda_p = 0.0363 \quad \beta_p = 1.1584$	PLP $\lambda_r = 0.4278 \quad \beta_r = 1.0560$
3	PLP $\lambda_p = 0.0715 \quad \beta_p = 1.1254$	PLP $\lambda_r = 0.2313 \quad \beta_r = 1.1133$

Note: These models were built with data of year 2005

Table 4.3. Pereira system - Input and service processes

Zone	$m_{tr}$ [Hours]	$m_{td}$ [Hours]	$m_{tw}$ [Hours]	$C$ [%]	$ma(t)$ [%]
1	2.08	4.13	2.05	49.63	49.11
2	1.53	2.00	0.47	23.50	21.74
3	1.72	2.79	1.07	38.35	34.53

Table 4.4. Pereira system – Repair process performance

- For the Pereira and Pasto systems, the input and repair processes are non-stationary with positive tendency. This means although the reliability of the components is deteriorating, the repair process is adjusting to follow the increasing pattern of failures arrivals.
- For the Casanare system, the input and repair processes are stationary but they do not correspond to the HPP.

Zone	Input process [Failures/hour]	Service Process [Repairs/hour]
1	Weibull RP $\lambda_p = 0.1814$ $\alpha_p = 0.3133 \beta_p = 0.7547$	Weibull RP $\lambda_s = 0.2732$ $\alpha_s = 0.5150 \beta_s = 0.6611$
2	Weibull RP $\lambda_p = 0.1971$ $\alpha_p = 0.4492 \beta_p = 0.6285$	Weibull RP $\lambda_s = 0.3086$ $\alpha_s = 0.5179 \beta_s = 0.7001$
3	Weibull RP $\lambda_p = 0.2175$ $\alpha_p = 0.3917 \beta_p = 0.7154$	Weibull RP $\lambda_s = 0.3125$ $\alpha_s = 0.6668 \beta_s = 0.5772$

Notes:

1. These models were built with data of years 2004-2006

2. The Weibull density function is defined as  $f(t) = \alpha \beta t^{\beta-1} \exp(-\alpha t^\beta)$

Table 4.5. Casanare system – Input and repair processes

Zone	$mttr$ [Hours]	$mtod$ [Hours]	$mtw$ [Hours]	$C$ [%]	$ma(f)$ [%]
1	3.66	19.72	16.06	81.44	68.0
2	3.24	18.22	14.98	82.22	65.0
3	3.19	23.41	20.22	86.37	70.0

Table 4.6. Casanare system – Repair process performance

Zone	Input process [Failures/hour]	Service Process [Repairs/hour]
1	PLP $\lambda_p = 0.5589 \beta_p = 1.0464$	PLP $\lambda_s = 1.6853 \beta_s = 1.0189$

Note: These models were built with data of year 2006

Table 4.7. Pasto system – Input and service processes

Zone	$mttr$ [Minutes]	$mtod$ [Minutes]	$mtw$ [Minutes]	$C$ [%]	$ma(f)$ [%]
1	32.34	53.49	21.15	39.54	37.10

Table 4.8. Pasto system – Repair process performance

- The performance of the repair process is directly connected with the size (area) of the service territory. For the Pereira and Casanare systems, the worst indices correspond to zones (service territories) with highest areas. The effect of remedial

actions proposed to reduce outage durations can be tested with this methodology.

- A low congestion or traffic intensity does not mean a low waiting time or consequently a low outage duration.
- The results of mean outage time correspond to those values observed during operation of the studied systems.

#### 4.6. Conclusions

The repair process performed in each service territory of a power distribution system is a queuing system. Thus, it has to be modeled using queuing models, not as part of component reliability models—the traditional approach applied in reliability assessments. Also, it is not realistic to apply the deterministic criteria  $n - 1$  for reliability assessments of this kind of system, as a failure can occur independently if the previous failure has been or not repaired.

As shown in the examples, the input and service of the repair process of a power distribution system are not necessarily HPP; they can be RP or NHPP, and for this reason, the system reliability assessment has to be performed by means of a sequential Monte Carlo simulation. The approach presented here, which considers stationary and non-stationary SPP for the failure and the service processes, is very different from traditional queuing modeling that assumes that these processes are Markovian (HPP) or semi-Markovian (one HPP and the other an RP).

The index better reflecting the performance of the repair process is the waiting time. A low congestion or traffic intensity does not necessarily mean a low waiting time or consequently a low outage duration. The proposed methodology explicitly evaluates the

performance of the repair process performed in a power distribution system and gives an analytical base for the optimal scheduling of the repair resources, in accordance with the failure process generated by the components and the targets for reliability indices.



## CHAPTER 5

# RELIABILITY ASSESSMENT OF A POWER DISTRIBUTION SYSTEM

The aim of this chapter is to show how the methodology of reliability assessment of a power system using SPP modeling and a sequential Monte Carlo simulation works. This is the first stage of development of a more comprehensive method that will be applied to assess the vulnerability of a composite system.

### 5.1. Introduction

Although methods to take aging and limited resources into account have been developed in the general reliability field and in queuing theory, these methods are not currently in widespread use in power distribution reliability assessment.

With regard to aging, published studies can be categorized as follows:

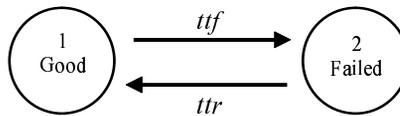
1. Studies at the component level: Aging is studied for given populations of components. This is the category with the most publications. Studies in this category can be subcategorized into:

- Those which model aging and external events that increase the failure rate.
  - Those which link reliability and maintenance, and determine an optimal preventive maintenance strategy to improve the components' reliability. The limitation of this type of studies is that they do not directly connect the components' reliability condition to customer reliability indices.
2. Studies at the system level: few papers have been written on the effect of component aging on load point reliability indices.

Regarding the repair process performed in power systems, it has been traditionally included in the reliability model of the components. This implies that repair resources are assumed unlimited, which is unrealistic.

## 5.2. Traditional Component Modeling

Figure 5.1 shows the basic and most popular reliability model for repairable components. This model is defined by the probability distributions of time to failure ( $t_{tf}$ ) and time to repair ( $t_{tr}$ ).



**Figure 5.1.** Two-state component reliability model

A probability distribution model always refers to a stationary process; hence, it always produces constant statistics (mean, variance). This means that, no matter what kind of distribution is

used for the reliability model, the expected number of failures that occur and the expected number of repairs that can be performed do not change as time passes. In other words, failure and repair intensities remain constant.

This kind of modeling has the following limitations:

1. Aging is not considered: Constant failure intensity means the component reliability does not improve or deteriorate, i.e., it is in its useful life period. This also means the component is under “renewal” because every time it fails, it is returned to an “as new” state. For this reason, this model is also known as an “alternating renewal process.”
2. Repair resources are unlimited: As the repair process is included in the component model, it is assumed that every time the component fails a crew is available to repair it; hence, repair time only depends on the particular actions taken to fix each type of component. Constant repair intensity means the performance of repair teams is not affected by internal or external factors. However, in real life, a crew has to fix all failed components located in its service territory. This means that some of them may have to wait while the ones that failed before are repaired. Also, crew performance is affected by external factors, such as weather and traffic, and also by internal factors, such as available tools and skills.

These limitations are also present in models with more than two states and in the simplified modeling that represents a repairable component as a block defined by means of a constant failure rate ( $\lambda$ ) and a mean repair time ( $r$ ).

### 5.3. Methods in Widespread Use for Reliability Assessment of Distribution Networks

#### 5.3.1. The Homogeneous Markov Process

If the failure and repair models of every component are exponential, the system model is a homogeneous Markov process. The mathematical model of a system with  $n$  components is the set of ordinary differential equations:

$$[\dot{P}]^t = [M]^t [P]^t, \quad (5.1)$$

where  $[P]$  is a row vector with the probabilities of the  $n * n$  states and  $[M]$  is the stochastic matrix of transition intensities between states.

Although this approach is very appealing because it gives analytical solutions, it has the following disadvantages:

1. For systems with many components, such as distribution networks, there is a huge number of system states. This makes its application a cumbersome task.
2. Repair times of power distribution components are, in general, log-normally distributed.
3. Loads are considered constant.

This method can include common mode outages and loss of component criteria of any order. However, some applications restrict its application to the  $n - 1$  case.

#### 5.3.2. Device of Stages

If the distribution of the failure process or of the repair process (or of both) is not exponential, the previous method does not

apply. For some pairs of distributions (exponential-lognormal, exponential-Weibull), this model can be transformed into a homogeneous Markov model by means of the device of stages method. Although this method solves the problem of non-exponentially distributed repair times, the resulting component model has more than two states, a fact that increases the dimension of the set of system equations. Thus, the first and third disadvantages of the previous method also hold for this one.

### 5.3.3. Simplified Method of Blocks

In this method, the system is represented as a network of components which are connected in series or parallel. It is also known as “minimal cut set method.” Each component is represented by a block with a constant failure rate ( $\lambda$ ) and a constant mean repair time ( $r$ ). A sequential reduction of series and parallel components can be used to solve the network of blocks.

As this method is derived from the homogeneous Markov process, it inherits disadvantages 2 and 3 of that method and adds the following:

1. It only gives expected values of the load point indices of failure frequency, mean time to repair and unavailability.
2. It does not consider the loss of more than one component.

### 5.3.4. Analytical Simulation

This method is an extension to the simplified method of blocks. It determines the fault contribution of each component and their impact on load points by means of the enumeration of each possible system state. In this method, each component is represented by the same parameters used in the simplified method of blocks.

Although this method can handle the loss of more than one component, it retains disadvantages 1, 2 and 3 of the Markov method and disadvantage 1 of the simplified method of blocks.

### 5.3.5. The Monte Carlo Simulation

This method can include any kind of distributions, time-varying loads, unbalanced conditions, common mode outages, etc. Disadvantages of this method are:

1. The high computational time required.
2. Multiple analyses on the same system could produce slightly different results.
3. It can overlook rare but important system states because it is not an enumerative method.

Disadvantages 1 and 3 can be mitigated by applying variance reduction techniques or importance sampling.

Although this method can include loss of component criteria of any order, it is often restricted to the  $n - 1$  case.

## 5.4. Methods for System Reliability Assessment that Can Include Aging

### 5.4.1. Manual Approach

In the methods listed in the previous section, component failure intensities are manually changed, for example, from their current values to the ones they are expected to have in the future or under a scenario where a given preventive maintenance strategy is applied. Scenarios for improvement and deterioration of the repair intensities can also be included.

#### 5.4.2. The Non-Homogeneous Markov Process

This method can include improvement and deterioration in the failure and repair processes. However, because the repair process is included in the reliability model of the component, the assumption is still made that unlimited repair resources are available. This model has disadvantages 1 and 3 mentioned for the homogeneous one. Solutions to this kind of process are numerical. However, this method has problems adjusting the operating times and of tractability of some types of time-varying rates [6].

#### 5.4.3. Stochastic Point Processes

In the general reliability field, the modeling of the failure process of repairable components by means of SPP started in the 1960s. The procedures for applying this kind of modeling are now well established and applications are mainly focused on reliability analysis of component populations.

In the power systems field, before the year 2000, applications focused on the reliability of component populations. For example, Schilling et al. studied thermal generators [12], Kogan and Jones studied underground cables [11], and Kogan and Gursky studied transmission towers [10].

In 2000, Stillman [13] applied this kind of modeling to the reliability of a distribution network considering it as a whole entity and showed results for rural and urban systems. He extended the method to distribution feeders, but again treating them as a whole entity [14]. In 2004 Balijepalli, Venkara and Christie [3] used this kind of modeling to obtain the distributions of load point indices of a real system. They used RP models for failures and repairs. Failure models were obtained from data but repair models were assumed.

## 5.5. Proposed Methodology

### 5.5.1. Modeling of Component Failure Processes

Component failures and common mode failures are modeled as independent failure sources using PLP or RP. When a PLP model is used:

- if  $\beta > 1$  the component is aging;
- if  $\beta = 1$  the component is in its useful life or as new; and
- if  $\beta < 1$  the component reliability is improving.

This kind of modeling implies that repairs are minimal, i.e., they only return the component to the operating state without improving or deteriorating its reliability condition.

If a component has two or more operating states, its failure process is represented by an SPP for each one. For example, a power transformer with operating states 100 MVA (ONAF) and 80 MVA (ONAN) requires two SPP failure models.

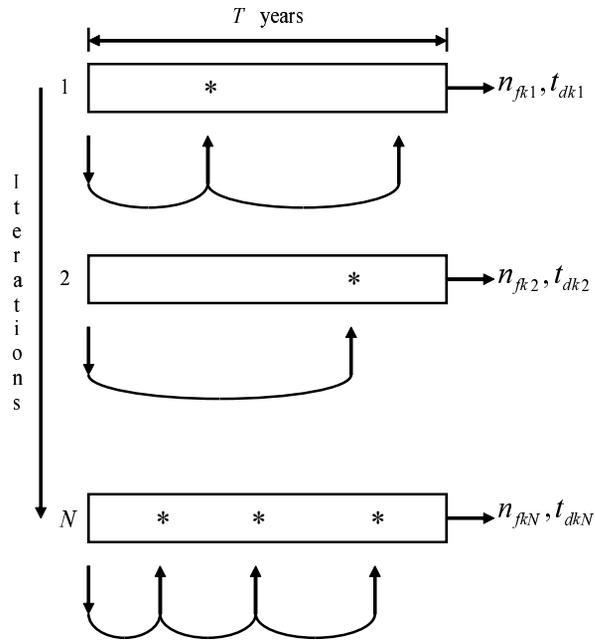
To obtain a component failure model, a sample of arrival or inter-arrival failure times is needed. This process includes failures caused by aging and other external factors like weather. A common approach in distribution networks is to pool data by component class in order to obtain one model that represents each component in a class. When using NHPP all failure models have to be synchronized to the same time reference.

### 5.5.2. Modeling of Repair Process

The repair process performed in each service territory of the distribution network is modeled by the queuing. This kind of modeling is explained in Chapter 4.

### 5.6. System Reliability Assessment

System operation is observed artificially for a period  $T$  of one or more years of interest by means of a sequential Monte Carlo simulation algorithm. As Figure 5.2 shows, a simulation consists of  $N$  iterations or artificial observations of system performance under a scenario defined by topology, expansion/upgrading, and forecasted demand.



**Figure 5.2.** Simulation procedure

The sequence of component failures and repairs is generated for each iteration; this is depicted in Figure 5.2 by an asterisk (\*) that indicates when a component fails and is repaired.

For a load point  $K$ , the samples of output variables such as the number of failures ( $n_{fk}$ ) and the down time ( $td_k$ ) allow the computation of reliability indices. Two stopping rules may be used

for the simulation: a fixed number of iterations, or the coefficient of variation of a load point index.

### 5.6.1. Iteration Procedure

1. Generate the failure processes of the components and the common mode failure processes for  $T$ .
2. Determine for each zone the failure process which consists of the superposition of the failure processes of all components located in the zone, including the common mode failures.
3. Generate a repair process. That is, for each failure  $f_i$  generate a  $ttr_i$ .

The arrival time of the first repair is:

$$tr_1 = tf_1 + ttr_1. \quad (5.2)$$

The arrival time of the next repair is determined in the following way:

- **Congestion:** If all crews are busy when failure  $i$  arrives, this failure has to wait until some crew finishes a repair  $j$  and fixes it.

$$tr_i = tr_j + ttr_i. \quad (5.3)$$

- **No congestion:** If a crew is free when failure  $i$  arrives, the repair for this failure starts immediately:

$$tr_i = tf_i + ttr_i. \quad (5.4)$$

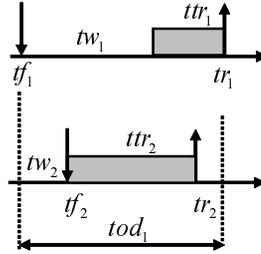
4. Calculate the outage duration:

$$tod_i = tr_i - tf_i. \quad (5.5)$$

5. Calculate the repair waiting time:

$$tw_i = tod_i + ttr_i. \quad (5.6)$$

6. Determine, for each load point, which failures affects its service continuity. The load point down time  $td$  for a given failure is equal to  $tod$  of that failure. If the repairs of two failures  $f_1$  and  $f_2$  affecting a load point overlap, combine them in the following way (see Figure 5.3):



**Figure 5.3.** Two overlapping failures affecting a load point

$$tf = \min(tf_1, tf_2); \quad (5.7)$$

$$tr = \max(tr_1, tr_2); \quad (5.8)$$

$$tod = tr - tf; \quad (5.9)$$

$$tw = \min(tw_1, tw_2); \quad (5.10)$$

$$ttr = tod - tw. \quad (5.11)$$

7. Determine the failure effect on load points: demand not served and affected customers.
8. For every load point accumulate the values of the output variables: failures, down time, load not served, etc.

### 5.6.2. Repair Process Indices

For each zone and sub-period of  $T$  (month, semester, etc.) compute the mean waiting time ( $mtw$ ), the mean outage duration ( $mtod$ ) and the congestion ( $C$ ) defined as:

$$C = \frac{mtw}{mtod} \cdot 100\%. \quad (5.12)$$

### 5.6.3. Load Point Indices

For each load point  $k$  and sub-period of  $T$  (month, semester, etc.) compute the adequacy indices. For example:

- Expected failure frequency:

$$\lambda_k = \sum_i^N \frac{n_{fki}}{N}. \quad (5.13)$$

- Mean time to repair:

$$r_k = \frac{\sum_{i=1}^N t_{dki}}{\sum_i^N n_{fki}}. \quad (5.14)$$

## 5.7. Example

Consider the overhead rural distribution system shown in Figure 5.4. For maintenance purposes, this system is split into two zones, each assigned to a crew.

Table 5.1 shows the components' reliability data. Most of this data is taken from surveys performed in a Colombian system during the period 2000-2007 [19, 20, 21, 22, 24, 25]. Data for busbars, common mode failures, and secondary distribution are typical values.

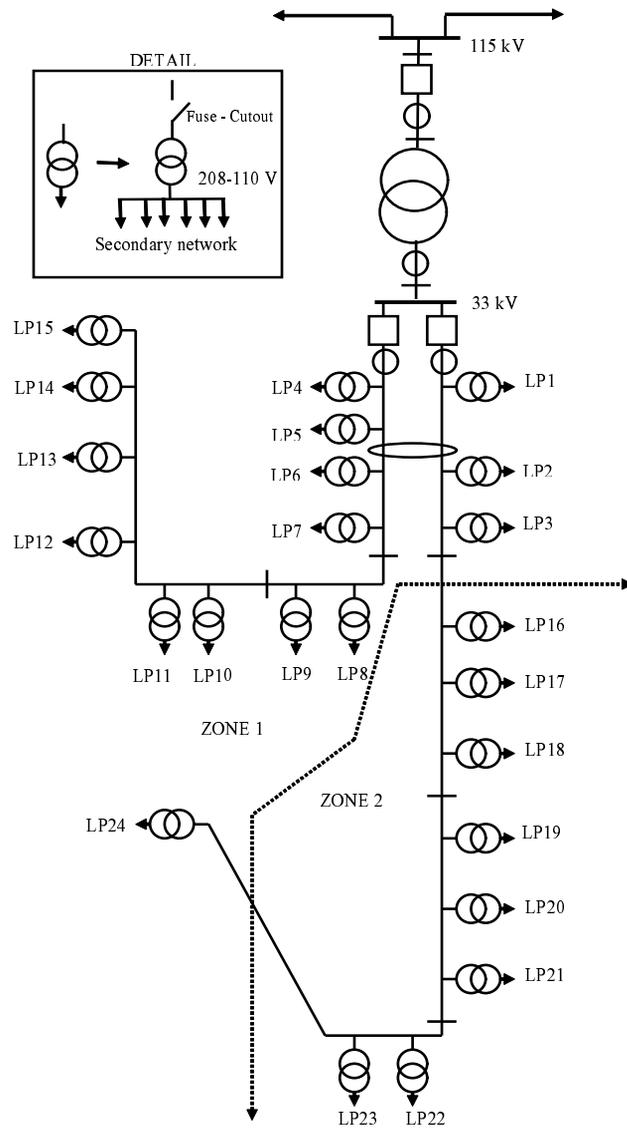


Figure 5.4. Test system

Table 5.2 shows the models applied for system reliability assessment. Case 1 assumes that all components are in their useful

Component	$\lambda_{\infty}$ : Sustained failure rate	
115/33 kV power transformer	0.1500	Failures/year
115 kV disconnector	0.1851	Failures/year
115 kV SF6 circuit breaker	0.2780	Failures/year
115 kV CT	0.0215	Failures/year
33 kV and 115 kV busbars	0.0200	Failures/year
33 kV overhead feeder	1.3375	Failures/km-year
Feeder common mode (15%)	0.13375	Failures/km-year
Distribution transformer	0.0064	Failures/year
Cut-out	0.1388	Failures/year
33 kV CT	0.0151	Failures/year
33 kV SF6 Disconnector	0.0569	Failures/year
33 kV SF6 Circuit breaker	0.2033	Failures/year
Secondary distribution	8.6360	Failures/year

Table 5.1. Component failure data

life and case 2 assumes that all components are aged. The repair models consider a coefficient of variation of 20%.

Case	Component failure models [Failures/year]	Zone repair models [Repairs/year]
1	HFP $\lambda(t) = \lambda_{\infty}$	Log-normal RP $\lambda(t) = 2190$
2	PLP $\lambda = \lambda_{\infty}$ $\beta = 1.5$	Log-normal RP $\lambda(t) = 1460$

Table 5.2. Study cases

The reliability assessment is performed for a period of 3 years. Tables 5.3 to 5.6 show the results obtained for each zone with 250 iterations.

Semester	Failures	$mttr$ [Hours]	$mtod$ [Hours]	$mtw$ [Hours]	Congestion [%]
1	87.5400	4.0156	4.2018	0.1863	4.4327
2	87.5280	3.9938	4.1693	0.1754	4.2074
3	88.8280	3.9997	4.1774	0.1778	4.2556
4	86.8800	4.0050	4.1886	0.1836	4.3824
5	86.9320	3.9989	4.1718	0.1729	4.1433
6	87.3240	4.0037	4.1940	0.1903	4.5364

Table 5.3. Results for Zone 1 – Case 1: No component aging

Semester	Failures	$mttr$ [Hours]	$mtod$ [Hours]	$mtw$ [Hours]	Congestion [%]
1	47.3480	5.9964	6.2120	0.2156	3.4713
2	47.3000	6.0184	6.2123	0.1939	3.1219
3	47.7440	6.0004	6.2094	0.2090	3.3680
4	46.9640	6.0073	6.2140	0.2067	3.3264
5	47.7400	6.0240	6.2463	0.2223	3.5590
6	47.5920	5.9850	6.2019	0.2169	3.4971

**Table 5.4.** Results for Zone 2 – Case 1: No component aging

Semester	Failures	$mttr$ [Hours]	$mtod$ [Hours]	$mtw$ [Hours]	Congestion [%]
1	61.8400	3.9906	4.1439	0.1533	3.6990
2	113.3920	3.9990	4.2411	0.2421	5.7081
3	146.9400	4.0024	4.3262	0.3238	7.4842
4	174.7360	4.0093	4.4030	0.3937	8.9418
5	196.9560	4.0003	4.4504	0.4500	10.1120
6	217.5400	4.0048	4.5181	0.5133	11.3604

**Table 5.5.** Results for Zone 1 – Case 2: Aged components

Semester	Failures	$mttr$ [Hours]	$mtod$ [Hours]	$mtw$ [Hours]	Congestion [%]
1	33.2920	6.0071	6.1863	0.1792	2.8963
2	60.6000	6.0024	6.3055	0.3031	4.8069
3	79.5080	5.9981	6.3660	0.3679	5.7799
4	94.3040	6.0080	6.4945	0.4885	7.5215
5	105.5400	6.0024	6.5307	0.5282	8.0886
6	117.4520	5.9977	6.5887	0.5910	8.9701

**Table 5.6.** Results for Zone 2 – Case 2: Aged components

These results show that the repair process indices  $mtw$ ,  $mtod$  and  $C$  increase over time if aging is present and the repair resources remain constant.

As shown in Figure 5.6, after 3 years,  $mtw$  in both zones is almost three times the value obtained in the case where aging is not considered.

As can be seen in Figures 5.5 and 5.7, the load point reliability index estimation shows similar results. After three years, the val-

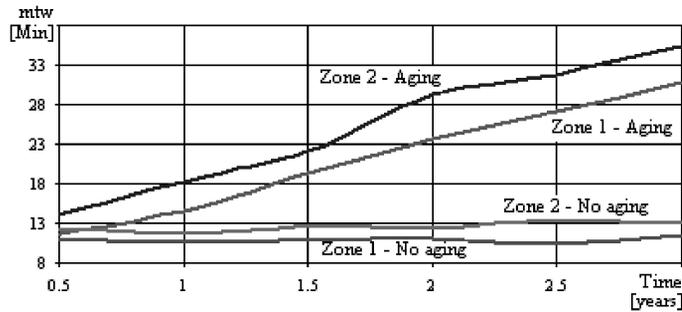


Figure 5.5. Mean waiting time for a repair

ues of  $\lambda$  and  $mtw$  range from 2.4 to 2.8 times the values obtained in the case where aging is not considered. This difference extends to other indices, such as SAIFI, CAIFI, SAIDI, CAIDI and EENS.

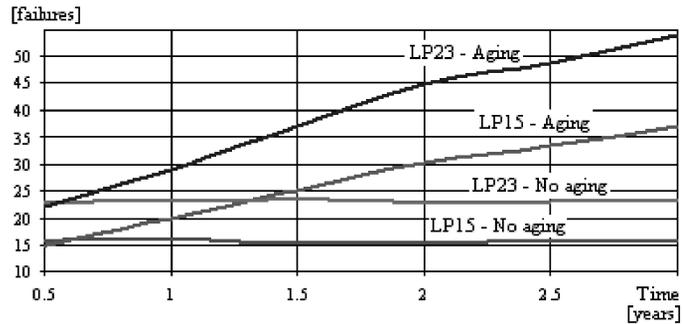
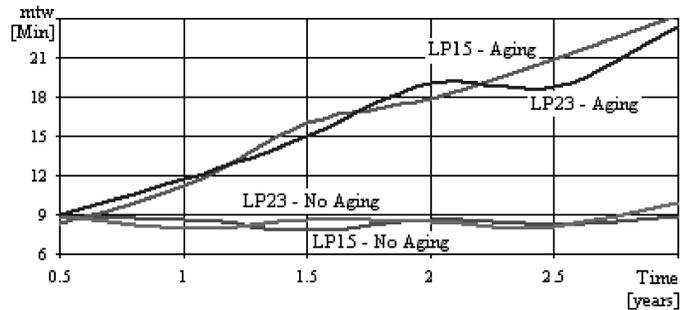


Figure 5.6. Failure frequency on two load points

### 5.8. Conclusions

SPP theory is an excellent modeling tool for reliability assessments of power systems: it allows the inclusion of component aging and the consideration of limits on repair resources, two important factors that cannot be represented by the traditional modeling and methods.



**Figure 5.7.** Mean waiting time for a repair on two load points

The repair process performed in each zone of a power system is a queuing system. Thus, it has to be modeled as such, not as part of component models. The input and service process of this system can be RP or NHPP, and, for this reason, the system reliability assessment has to be performed by means of a sequential Monte Carlo simulation.

The proposed methodology evaluates the performance of the repair process performed in a distribution network and gives an analytical basis for the optimal scheduling of the repair resources, in accordance with the failure process generated by the components and the targets for reliability indices. This kind of analysis is required even for power systems where aging is not a matter of concern.

Results obtained using the proposed methodology show that congestion, waiting time for repair and customer outage time, increase dramatically if components are used far beyond their useful life and repair resources are not increased to match the rate of component failures.



## CHAPTER 6

# RELIABILITY ASSESSMENT OF A PROTECTIVE SCHEME

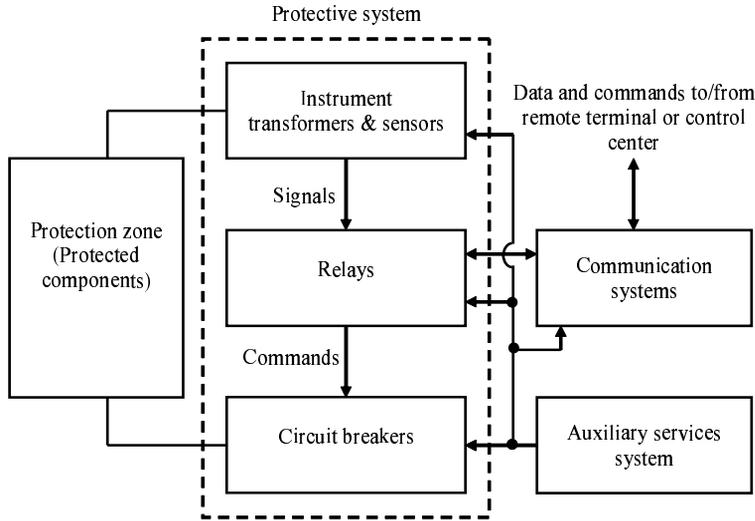
The aim of this chapter is to show the way protective systems are incorporated into the proposed approach of modeling. Basically, a method for the assessment of protective schemes is developed here. It will be applied later in Chapter 8 to obtain a condensed model of failures to operate of protective systems. The condensed model will be used in the assessment of the power system.

The content of this chapter is taken from the paper “Reliability Assessment of Protective Schemes Considering Time-Varying Rates,” by Zapata, Kirschen, Torres and Ríos [28].

### 6.1. Introduction

The mission of a protective system (PS) is to detect abnormal operating conditions in the protection zone (PZ) to which it is assigned and to take actions that guarantee power system safety and security and safeguard investment in power system assets.

Figure 6.1 shows the main types of protective system components (PSC).



**Figure 6.1.** Components of a protective system

A protective scheme is the optimal combination of PSC which allows the PS to perform its mission with a specified level of reliability. Reliability refers to the degree of certainty that the PS will perform correctly. It combines the redundancy and diversity aspects of the PSC.

Due to the critical mission assigned to the PS and the fact that maloperations can spark a sequence of cascading outages that could lead to a catastrophic event such as a blackout, PS reliability is a matter of utmost importance. This fact has long been understood and has been studied from several points of view.

Reliability studies of PS can be classified into the following categories:

1. Studies at the component level focus on a given component of the PS, for example, a relay.
2. Studies at the PS level focus on protective schemes at the terminals of the PZ.

3. Studies at the power system level focus on the effects of PS failures on power system reliability.

The study presented here focuses on studies of the second type. They are helpful for:

- comparing design alternatives;
- assessing the effect of incorporating PSC with various levels of reliability; and
- evaluating the impact of different preventive maintenance strategies.

## 6.2. Problem Statement

Reliability assessments of protective schemes have been traditionally performed under the assumption that PSC failure and repair processes are stationary; this implies constant failure and repair rates, constant probabilities of failure or constant availabilities. Hence, the mathematical methods used for this task are those that work under this assumption; for example, event trees, failure trees, reliability blocks and homogeneous exponential Markov chains.

Although stationarity has long been a common assumption in power system reliability, its relevance should be carefully re-examined because of the growing importance of factors such as aging, improvement/decrease in preventive maintenance and repair resources, and the recognition that failure and repair rates can be time-varying functions. If stationarity is no longer a valid assumption, the application of the mathematical methods mentioned above is no longer valid. This chapter thus presents a method on Stochastic Point Process (SPP) theory because this approach can handle time-varying rates.

### 6.3. Failure Modes of a Protective System

A PS can take two kinds of actions: disconnection and connection of the PZ. These actions can arise automatically, due to abnormal operating conditions in the PZ, or manually, due to intentional or unintentional orders given by an operator. These actions are materialized through the opening and closing of the circuit breakers associated with the PZ. Requests to the PS to come into action can thus be calls to open (CTO) or calls to close (CTC).

A PS operates correctly and appropriately if it does not fail when it is called to operate and does not operate when this is not required. The basic PS failure modes are failures to operate, which include failures to open (FTO) and failure to close (FTC), and false operations, which include false openings (FO) and false closings (FC). Failures to operate include those situations where the opening or closing takes more than the specified time.

Failures of PSC are classified here in accordance with their potential effect on PS operation, viz., as FTO, FTC, FO and FC. The term “potential” is used because the final effect of a PSC failure on the PS operation depends on the configuration of the protective scheme. Another type of PSC failure is the knocking down (KND) which could lead to a situation where the PS does not operate. All these failure modes do not necessarily apply to every PSC.

### 6.4. Protective System Reliability Indices

#### 6.4.1. Reliability

Reliability refers to the degree of certainty that the PS will perform correctly. It is measured as the ratio of wanted openings

and closings which were performed successfully to the number of exposures:

$$R = \frac{(CTO - FTO) + (CTC - FTC)}{CTO + CTC + FO}. \quad (6.1)$$

#### 6.4.2. Dependency

The term *dependency* refers to the degree of certainty that the PS will perform correctly when it is called upon to operate. It is measured as the ratio of wanted openings and closings which were performed successfully to the number of calls to operate:

$$R = \frac{(CTO - FTO) + (CTC - FTC)}{CTO + CTC}. \quad (6.2)$$

#### 6.4.3. Security

Security refers to the degree of certainty that the PS will not produce false operations. It is measured as the ratio of wanted openings which were performed successfully to the number of wanted and unwanted openings which were performed:

$$S = \frac{(CTO - FTO)}{(CTO - FTO) + FO}. \quad (6.3)$$

### 6.5. Protection Zone Reliability Index

PS maloperations affect the PZ service continuity; thus, they are reflected in the PZ operational reliability:

$$U_o = \sum \frac{u_i}{T}, \quad (6.4)$$

where  $u_i$  is the unavailability of the PZ due to an outage  $i$ .

## 6.6. Proposed Method

### 6.6.1. Modeling

Each failure mode that applies to the PZ is represented by means of an SPP model. These modes are: permanent faults, temporary faults, and common mode faults between PZ and PS. Each failure mode that applies to a given PSC is represented by means of an SPP model. To obtain these models, failure data is divided based on the failure mode, and the resulting sample data for each failure mode is fitted to an SPP.

An SPP is fitted to the repair sample data corresponding to each failure mode of the PSC and the PZ. It is assumed that repair actions are perfect, i.e., that they effectively eliminate failures and do not introduce new ones.

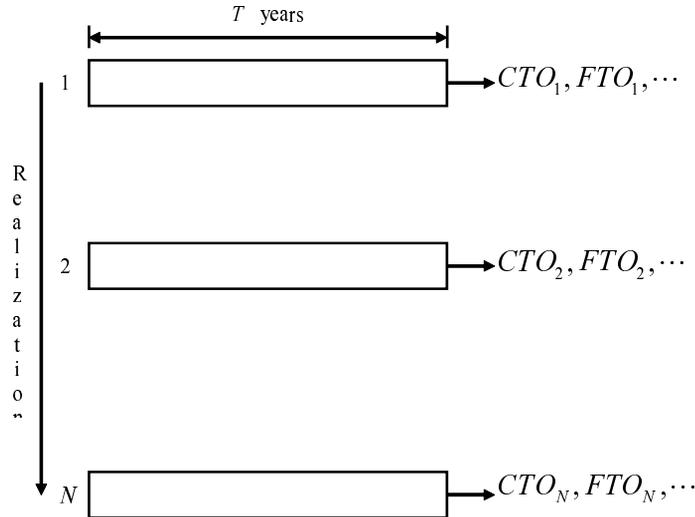
Preventive maintenance on the PZ and its PS include the actions performed by maintenance personnel and the auto-diagnostic functions (self-check and monitoring) incorporated in some PSC, such as relays. The time of occurrence of the events of these processes is deterministic because they are programmed to occur at fixed intervals; thus, they are generated using their yearly frequency. Their duration is random and so it is modeled by means of an SPP. Since these processes are not perfect in their function of finding PSC failures, this feature is represented by means of  $E$ , the probability of finding a PSC failure.

### 6.6.2. Reliability Assessment Procedure

The operation of the PS associated to a PZ is observed artificially for a period  $T$  of one or more years of interest by means of a procedure of a sequential Monte Carlo simulation (MCS).

The application of MCS is justified by the fact that it is the only method that can manage all probabilistic models of any type, stationary and non-stationary, and also because it easily incorporates all actions which happen during the operating sequence of a PZ and its PS, such as failures, repairs, maintenances, and self-checks.

As depicted in Figure 6.2, a simulation consists of  $n$  artificial observations of PS performance during  $T$ , under a scenario defined by the protective scheme configuration, the failure and repair rates and the strategy for preventive maintenance. The output of a realization is the set of variables which allow computing the indices of the PS model, viz.,  $CTO$ ,  $FTO$ ,  $CTC$ ,  $FTC$  and  $FO$ .



**Figure 6.2.** General procedure of the reliability assessment algorithm

### 6.6.3. Procedure Inside a Realization

The procedure inside a realization is depicted in Figure 6.3. Each downward arrow symbolizes the occurrence of a failure or maintenance in a PZ with a PS composed of PSC. The steps of this procedure are:

1. Generate the failure process of PZ ( $f_1 f_2 \dots f_n$ ).
2. Generate the failure processes corresponding to each PSC.
3. Generate the process of preventive maintenance that requires the disconnection of PZ ( $m_1 m_2 \dots m_n$ ).
4. Generate the processes of self-check, monitoring and preventive maintenance on PSC that do not require the disconnection of PZ.
5. For each  $f - i$  or  $m_i$  analyze if the PS operates correctly for a CTO and a CTC, i.e., observe if PSC failures have occurred before each call to operate and determine if they lead to a PS failure to operate. Tie sets corresponding to the request (CTO, CTC) and its origin (automatic, manual) are used to determine PS success or failure. For FTO and FTC it is assumed that PSC and PZ repairs can be performed simultaneously; thus, PSC failures only add unavailability to the PZ when they last more than PZ repairs.
6. For each PSC false opening generated whilst the PZ is in the operating state, determine if the PS produces a trip. This requires evaluating the tie sets which guarantee the trip can be performed. Also analyze if the PS operates correctly when CTC.
7. Repeat steps 1 to 6  $n$  times.

8. For each sub-period  $k$  (week, month, semester, year, etc.) of  $T$  compute the indices of the PS failure model. When using time-varying rates, reliability indices should not be computed for a single sub-period equal to  $T$  because variation is lost.

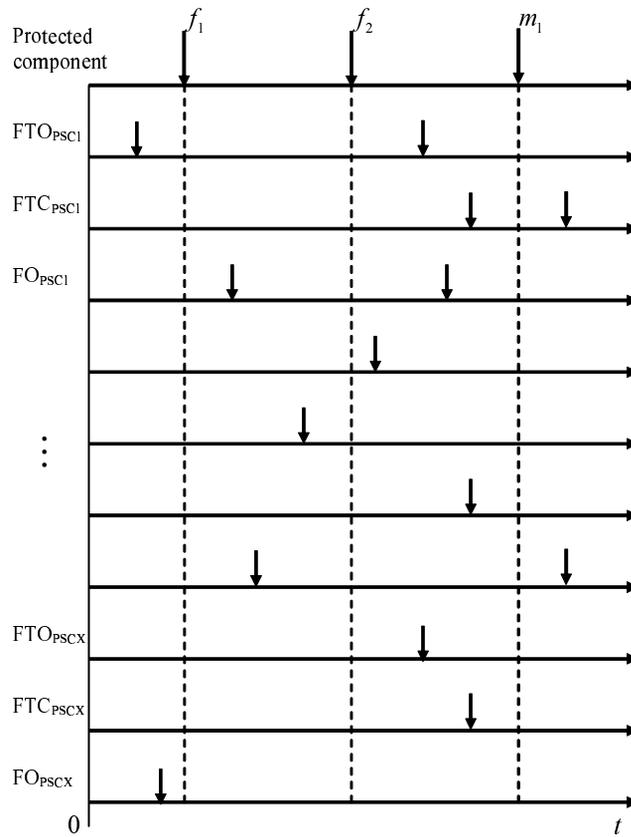


Figure 6.3. General procedure inside a realization



This PS has three circuit breakers (11, 12 and 13), two current transformers (21 and 22), an overcurrent relay (31), a differential relay (32), a Buchholz relay (33) and auxiliary services (41).

The following PSC are not shown in Figure 6.4 but included in the study: a 115 kV closing circuit (51), a 34.5 kV closing circuit (52), a 115 kV opening circuit (61) and a 34.5 opening circuit (62).

Tables 6.1 and 6.2 show the reliability data for the PZ and the PS, respectively.

$\lambda_F$	$r$
0.15 [failures/year]	2.00 [hours]

**Table 6.1.** Power transformer reliability data

Data for the opening/closing circuits were estimated from typical values; other data were estimated from indices obtained in several reliability surveys performed in Colombia [19], [16].

### 6.7.2. Study Cases

1. Failure processes of PZ and PSC are modeled as HPP with  $\lambda(t) = \lambda_F$ . Repair processes and preventive maintenance durations are modeled as normal RP with  $\lambda(t) = 1/r$ . There is only a preventive maintenance event per year with a mean duration of 12 hours.  $E = 80\%$  for FTO and FTC and  $E = 10\%$  for FO. This case reflects a situation where failure and maintenance processes are stationary.
2. The failure processes of components 11, 12, 13, 31 and 32 are modeled using a Power Law process with scale parameter  $\lambda$  equal to the values for  $\lambda_F$  shown in Table 6.2 and shape parameter  $\beta = 1.2$ . The failure process of these components is thus non-stationary with a positive tendency. Other models

PSC	KND	FTO	FTC	FO
11	$\lambda_F = 0.0278$	$\lambda_F = 0.0834$	$\lambda_F = 0.0834$	$\lambda_F = 0.0834$
12	$r = 2.00$	$r = 2.00$	$r = 2.00$	$r = 2.00$
13	$\lambda_F = 0.0204$	$\lambda_F = 0.0610$	$\lambda_F = 0.0610$	$\lambda_F = 0.0610$
	$r = 3.00$	$r = 3.00$	$r = 3.00$	$r = 3.00$
21	$\lambda_F = 0.0086$	$\lambda_F = 0.0011$	—	$\lambda_F = 0.0086$
	$r = 1.00$	$r = 1.00$	—	$r = 1.00$
22	$\lambda_F = 0.0060$	$\lambda_F = 0.0008$	—	$\lambda_F = 0.0060$
	$r = 1.00$	$r = 1.00$	—	$r = 1.00$
31	$\lambda_F = 0.0022$	$\lambda_F = 0.0033$	—	$\lambda_F = 0.0044$
	$r = 1.00$	$r = 1.00$	—	$r = 1.00$
32	$\lambda_F = 0.0054$	$\lambda_F = 0.0081$	—	$\lambda_F = 0.0108$
	$r = 1.00$	$r = 1.00$	—	$r = 1.00$
33	$\lambda_F = 0.0088$	$\lambda_F = 0.0132$	—	$\lambda_F = 0.0176$
	$r = 1.00$	$r = 1.00$	—	$r = 1.00$
41	$\lambda_F = 0.0183$	—	—	—
	$r = 8.00$	—	—	—
51 52	—	—	$\lambda_F = 0.0015$	—
	—	—	$r = 8.00$	—
61 62	—	$\lambda_F = 0.0015$	—	$\lambda_F = 0.0005$
	—	$r = 8.00$	—	$r = 8.00$

**Table 6.2.** Protective system reliability data

are the same as in case 1. This case reflects a situation of aging and no strategy for improving preventive maintenance.

3. The same as in case 2, but now preventive maintenance frequency is increased 100% each year. This case reflects a situation of improving preventive maintenance to reduce the effect of aging.

### 6.7.3. Results

Tables 6.3, 6.4 and 6.5 show the results for  $T = 3$  years and  $n = 10\,000$  realizations. Figures 6.5 to 6.7 show  $R$ ,  $D$  and  $S$  for the cases studied. Simulations lasted 0.47 hours, 5.34 hours and 8.81 hours for cases 1, 2 and 3, respectively.

Year	$R$	$D$	$S$	$U_o$
1.0–3.0	84.6754	93.7566	78.3917	0.1483

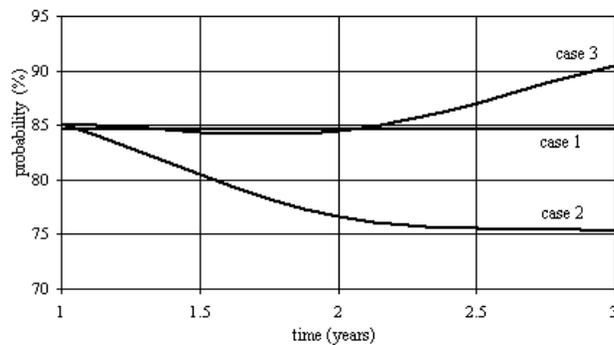
**Table 6.3.** Results for case 1 [%]

Year	$R$	$D$	$S$	$U_o$
1.0	85.1367	94.2183	78.7180	0.1483
2.0	76.6338	86.7812	70.4375	0.1510
3.0	75.3358	85.5957	69.0022	0.1522

**Table 6.4.** Results for Case 2 [%]

Year	$R$	$D$	$S$	$U_o$
1.0	85.1268	94.2235	78.6899	0.1483
2.0	84.5231	90.9193	83.3357	0.2882
3.0	90.5142	94.3660	90.9171	0.5632

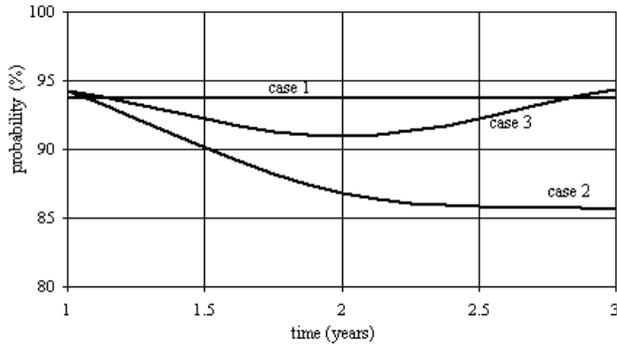
**Table 6.5.** Results for Case 3 [%]



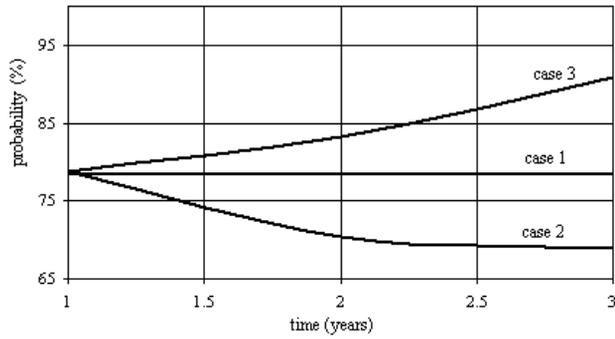
**Figure 6.5.** Reliability of the protective system

#### 6.7.4. Analysis of the Results

In the first case the PS reliability indices are constant because all PSC failure and repair processes are stationary; thus, it is only necessary to calculate them for one year.



**Figure 6.6.** Dependency of the protective system



**Figure 6.7.** Security of the protective system

In the second case, the presence of some aged PSC decreases the PS reliability indices. As can be seen in the results for  $U_o$ , the presence of some aged PSC increases the unavailability of the PZ.

Results for case 3 show how the improvement in preventive maintenance increases PS reliability even in the presence of aging; however, as can be seen in the results for  $U_o$ , this strategy decreases PZ availability. Thus, the analyst has to assess if the cost of PZ unavailability and additional maintenance pays the replacement of aged PSC.

Simulation times show how, as more details a reliability assessment includes, the longest the required simulation time is.

## 6.8. Conclusions

A new method for reliability assessment of protective schemes is presented in this work. Unlike traditional methods, it supports the consideration of time-varying failure and repair rates, and diverse maintenance strategies. However, the great improvement in modeling detail offered by this method has a price—the long computational time required by the simulation. Thus, its application is only recommended for those situations involving time-varying rates, for otherwise it is simpler and faster to apply the traditional methods.

This method can also be easily extended to reliability assessment of small portions of a power system such as substations.



## CHAPTER 7

# RELIABILITY ASSESSMENT OF A SUBSTATION

The aim of this chapter is to apply the method of stochastic point process modeling and a sequential Monte Carlo simulation to the assessment of a small portion of a power system. This assessment includes the protective system associated to each main power system component.

The content of this chapter is taken from the paper “Reliability Assessment of Substations Using Stochastic Point Processes and Monte Carlo Simulation,” by Zapata, Alzate and Ríos [18].

### 7.1. Introduction

Electrical power substations are the most critical parts of a power system, for it is there where the main power system components interconnect. A substation failure can produce the outage of many power system components, which can be disastrous for the system. For this reason, substation reliability is a matter of outmost importance.

Reliability of a substation depends on:

- The substation configuration, i.e., the arrangement of circuit

breakers or busbars.

- The reliability of substation components.
- The reliability of protective systems.

A substation reliability assessment evaluates the effect of these aspects on the service continuity of the main power system components connected to the substation.

## 7.2. Motivation

The following aspects motivated the development of the proposed method:

### 7.2.1. The Necessity of Considering Time Varying Rates

Reliability assessments of substations has been traditionally performed under the assumption that the failure and repair processes of substation components are stationary; it is expressed by means of constant event rates, constant probabilities of failure or constant availabilities.

This practice is also reflected in the mathematical methods that have been applied for this task: cut sets, reliability blocks, homogeneous Markov chains, fault trees, etc. However, nowadays the application of this assumption ought to be carefully examined because, due to factors such as aging, improvement/decrease on preventive maintenance and repair resources, the failure and repair rates of substation components can be time-varying functions.

In order to manage time-varying rates with the traditional methods, the analyst has the following options:

1. To manually change the input parameters (event rates, failure probabilities, availabilities) to the expected values for the

future years under several scenarios of improvement/deterioration of component reliability, preventive maintenance and repair process performance. However, this approach is not very accurate because event rates do not change in discrete steps but continuously.

2. To incorporate the functions which represent the time-varying rates into a non-homogeneous Markov process. However, this method has problems for adjusting the operating times and of tractability for some types of time-varying rates.

Thus, the proposal here is to model the failure and repair processes of substation components by means of SPP. It allows the utilization of time-varying rates in an easier way than in the non-homogeneous Markov chain method.

### **7.2.2. The Necessity of Including the Effect of Protective Systems**

It has been a common practice for reliability assessments of substations to only consider the primary plant components, i.e., the high-voltage ones, and assume the effect of items of secondary plant such as protective components, auxiliary services, communication systems, cablings, etc. are included in the reliability models of the high-voltage ones.

Regarding this practice, Dortolina et al. [5] pointed out that:

*“There are practical situations where it would be important to explicitly evaluate the influence of the protective relaying equipment on the overall substation reliability. These include: (i) Evaluating the effect of a given protective scheme on the reliability of different substation arrangements, and (ii) evaluating the effect*

*of the redundancy of the protective relaying equipment on the reliability of a given (and perhaps existing) substation.”*

This practice is in part justified by the fact that all analytic methods for reliability assessment that allow a detailed system representation require an exhaustive list of operating states; thus, if many components are considered, the amount of system operating states becomes huge. On the other hand, the proposal here is to perform the reliability assessment by means of a procedure of sequential MCS. This allows including as many components and operating conditions as the analyst wants and does not require an exhaustive list of system operating states.

### 7.3. Concept of Protection Zones

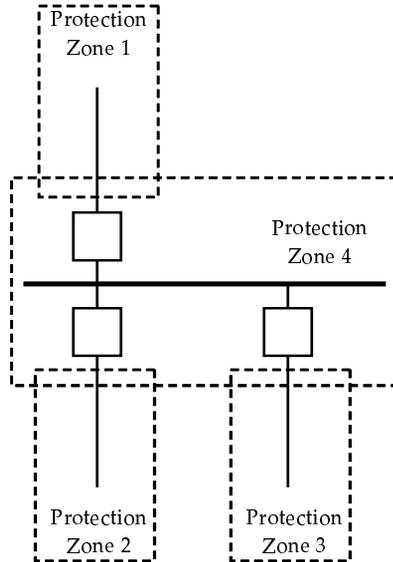
Each main power system component (transmission line, power transformer, reactive compensation, busbar) connected to a substation defines a protection zone (PZ); this concept is illustrated in Figure 7.1.

Each PZ has a protective system (PS) composed of several protective system components (PSC); PSC include circuit breakers, disconnectors, instrument transformers, relays, trip circuits, etc. Communication and auxiliary service systems can be part of each PS or shared by several of them.

### 7.4. Failure Modes of Protected Zones

Failures of PZ can be:

- (i) **Permanent:** Those failures that have to be repaired by maintenance personnel.



**Figure 7.1.** Protection zones associated to a single busbar substation

- (ii) **Temporary:** Those failures that disappear without taking any repair action; thus, the PZ is reconnected by means of an automatic reclosing action.

## 7.5. Common Mode Failures

Common mode failures are those that simultaneously affect PZ and PSC. Most of these kinds of failures are permanent.

## 7.6. Tie Sets

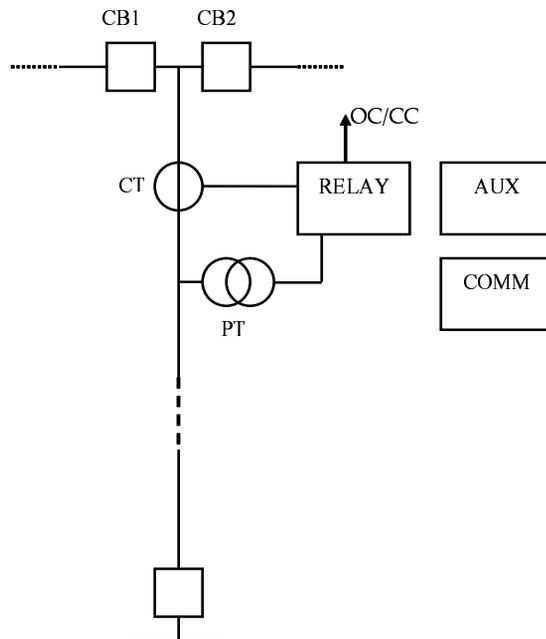
In the proposed method, every time a PS is called to operate, or a PSC produces a false opening, the PS outcome is determined using tie sets.

A tie set is a group of components which, when OK, guarantee the system can perform a given action; hence, they are connected in series from a reliability point of view.

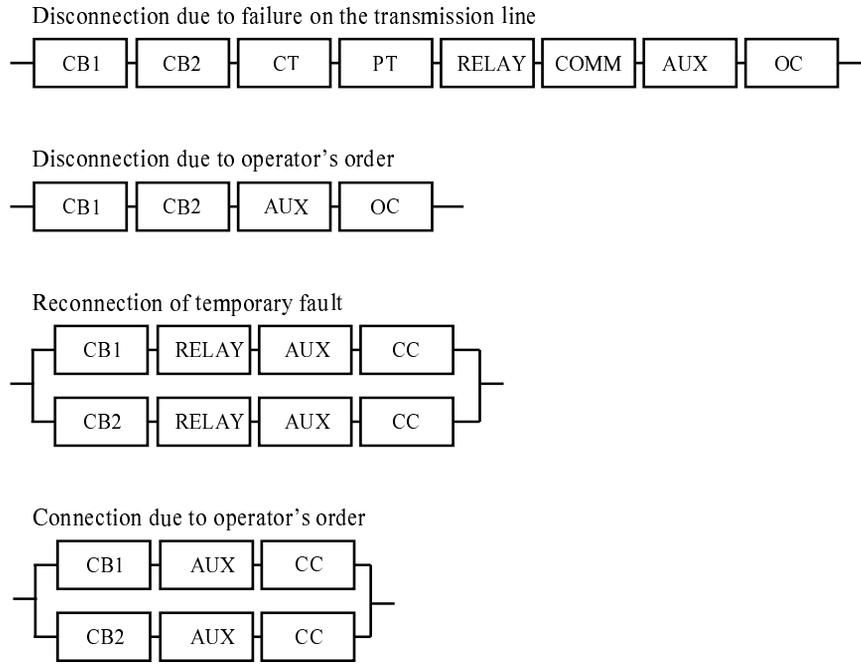
As there can be several paths or combinations of components that guarantee the system can perform a given action, there are several tie sets that are connected in parallel from a reliability point of view.

A system succeeds in performing a given action if at least one tie set is OK, and it fails when all tie sets are down.

To illustrate this, let us consider the PS shown in Figure 7.2. It includes two circuit breakers (CB1, CB2), instrument transformers (CT, PT), auxiliary services system (AUX), communication system (COMM), opening circuit (OC) and closing circuit (CC). The tie sets are shown in Figure 7.3.



**Figure 7.2.** Protective system of a radial transmission line



**Figure 7.3.** Tie sets for the PS of the transmission line shown in Figure 7.2

Tie sets are used in the proposed method because they have a data structure that can be easily codified in matrix form.

## 7.7. Proposed Method

### 7.7.1. Modeling

Each failure mode that applies to a PZ is represented by means of an SPP model. To obtain these models, PZ failure data is split up by failure mode and the resulting sample data for each failure mode is fitted to an SPP. Likewise, each failure mode that applies to a given PSC is represented by means of an SPP model. To obtain these models, failure data of each PSC is split up by failure

mode and the resulting sample for each failure mode is fitted to an SPP.

Each common mode failure is represented by means of an SPP model. Data samples to obtain these models are taken from PZ and PS failure data.

An SPP is fitted to the repair sample data corresponding to each failure mode. It is assumed that repair actions are perfect, i.e., they effectively eliminate failures and do not introduce new ones.

For FTO and FTC it is assumed that PSC and PZ repairs can be performed simultaneously; thus, PSC failures only add unavailability to the PZ when they last more than PZ repairs.

Preventive maintenance on PZ and PS include the actions performed by maintenance personnel and the auto-diagnostic functions (self-check and monitoring) incorporated in some PSC, such as relays. The time of occurrence of the events of these processes is deterministic because they are programmed in the form of fixed intervals; thus, they are generated using their yearly frequency. Their duration is random and so they are modeled by means of an SPP. These processes are not perfect in their function of finding PSC failures; this feature is represented by means of  $E$ , the probability of finding a PSC failure.

### 7.7.2. Reliability Assessment Procedure

The operation of each PZ and its associated PS is observed artificially for a period  $T$  of one or more years of interest by means of a procedure of sequential MCS.

A simulation consists of  $n$  artificial observations of the operating sequence of PZ and PS under a scenario defined by substation configuration, protective scheme configuration, failure and repair

rates, and strategy for preventive maintenance. FC are not considered because they do not affect PS operation.

### 7.7.3. Procedure Inside a Realization

The procedure inside a realization is depicted in Figure 7.4. Each downward arrow symbolizes the occurrence of an event of failure or maintenance in a PZ with a PS composed of  $X$  PSC.

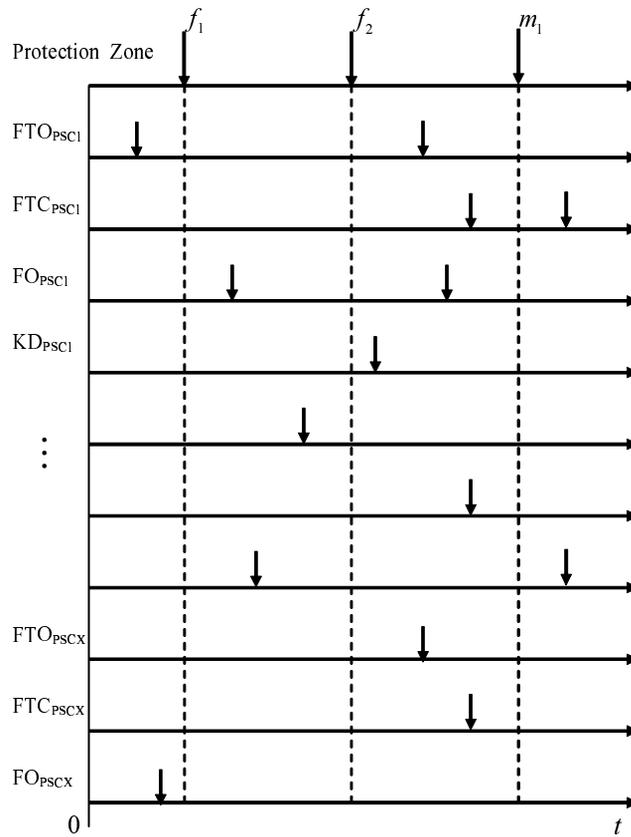


Figure 7.4. General procedure inside a realization

Steps are:

1. Generate the PZ failure process  $(f_1 f_2 \dots f_n)$ .
2. Generate the failure processes corresponding to each PSC.
3. Generate the process of preventive maintenance that requires the PZ disconnection  $(m_1 m_2 \dots m_n)$ .
4. Generate the processes of self-check, monitoring and preventive maintenance on PSC which does not require the PZ disconnection.
5. For each  $f_i$  or  $m_i$  analyze if the PS operates correctly when CTO and CTC; i.e., observe if PSC failures have occurred before each call to operate and determine if they lead to a PS failure to operate.
6. If the PS fails to operate, determine if the PS that give local back up operate correctly and the additional PZ that were disconnected. Determine the effect of PS failure in PZ availability.
7. For each PSC false opening that has been generated while PZ is in the operative state, determine if the PS produces a trip. This is performed evaluating the tie sets which guarantee that the trip can be performed. Also analyze if the PS operates correctly when CTC. Determine the effect of false opening in PZ availability.
8. Repeat steps 1 to 7  $n$  times or after reaching other stopping rule.
9. For each sub-period  $k$  (week, month, semester, year, etc.) of  $T$  compute the indices of the PZ. When using time-varying

rates, reliability indices should not be computed for a single sub-period equal to  $T$  because variation is lost.

#### 7.7.4. Detection of Failures by Preventive Maintenance

For each PSC failure that is present when these processes are performed, a uniform distributed random number  $U$  is generated. If  $U \leq E$ , it is detected; on the contrary, it remains undetected.

#### 7.7.5. Reliability Indices

Substation reliability is measured by means of indices related to the service continuity of each PZ. These are expected event rates, expected availability and expected unavailabilities.

Definitions:

$o$  : An outage event, i.e., a PZ disconnection.

$u$  : Down time due to an outage.

$f$  : A PZ outage due to a failure in this zone or in an upper hierarchical PZ.

$m$  : A PZ outage due to a preventive maintenance action in this zone or in an upper hierarchical PZ.

$fo$  : A PZ outage caused by a false opening originated in its own PS or in a one of an upper hierarchical PZ.

$bu$  : A PZ outage caused by a backup action taken by its own PS or by a one of an upper hierarchical PZ.

- Expected operational outage rate:

$$\lambda_o = \frac{\sum(f + m + fo + bu)}{T - \sum u}. \quad (7.1)$$

- Expected operational unavailability:

$$U_o = \sum \frac{u}{T}. \quad (7.2)$$

- Expected operational availability:

$$A_o = 1 - U_o. \quad (7.3)$$

The other expected event rates and unavailabilities are computed in the following way:

$$\lambda_e = \frac{\sum e}{T - \sum_{\forall u \in e} u}; \quad (7.4)$$

$$U_e = \sum_{\forall u \in e} \frac{u}{T}, \quad (7.5)$$

for  $e = f, m, fo, bu$ .

## 7.8. Example

### 7.8.1. Test System

In order to illustrate how the proposed method works, let us consider the air-insulated, single-busbar rural substation shown in Figure 7.5. It comprises switchyards for a sub-transmission line and two feeders. It is projected the addition of new switchyard for a third feeder.

Table 7.1 shows the PZ reliability data. It includes the mean failure rate  $\lambda$  and the mean repair time  $r$ .

Table 7.2 shows the PSC failure rates. PSC failure rates were estimated from data obtained in two reliability surveys performed in Colombia [16, 19, 21, 22, 25].

Failure rates for opening and closing circuits and repair times for all PSC were estimated from typical values. PZ data was assumed.

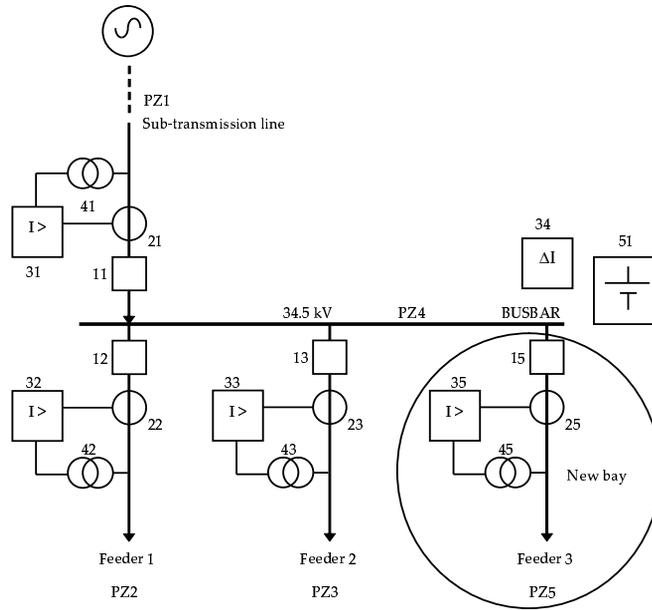


Figure 7.5. Test system

PZ	Permanent failures		Temporary failures	
	$\lambda$	$r$	$\lambda$	$r$
	[Failures/year]	[hours]	[Failures/year]	[minutes]
1	1.0	6.0	2.0	2.0
2	2.0	6.0	4.0	2.0
3	4.0	6.0	8.0	2.0
4	0.2	12.0	---	---
5	3.0	6.0	6.0	2.0

Table 7.1. Reliability data of protection zones

Component	KD	FTO	FTC	FO
Circuit breaker	0.0203	0.0610	0.0610	0.0610
Current transformer	0.0060	0.0008	---	0.0060
Closing circuit	---	---	0.0015	---
Opening circuit	---	0.0015	---	0.0005
Overcurrent relay	0.0022	0.0033	0.0022	0.0033
Differential relay	0.0054	0.0108	---	0.0108
Auxiliary services	0.0183	---	---	---

Table 7.2. Failure rates of protective system components [failures/year]

Preventive maintenance frequency is one event per year in each zone.  $E = 80\%$  for FTO and FTC, and  $E = 20\%$  for FO. Existing relays do not incorporate auto-diagnostic functions.

The mean duration of preventive maintenance events is  $r = 12$  hours.

It is assumed the PZ failure processes are stationary and that existing circuit breakers and relays are aged. All stationary failure processes are modeled by means of HPP with  $\lambda(t) = \lambda$ . The failure processes of aged components are modeled by means of a Power Law process with scale parameter equal to  $\lambda$  and shape parameter  $\beta = 1.2$ .

Repair processes and preventive maintenance durations are modeled as normal RP with  $\lambda(t) = 1/r$  and variance of 50%,  $r = 8$  hours for high-voltage switchgear, opening/closing circuits and auxiliary services and  $r = 4$  hours for protective relays.

### 7.8.2. Study Cases

1. A reliability assessment considering the real reliability condition of the components.
2. A reliability assessment considering that all PSC failure processes are stationary.

The study focuses on the substation reliability indices seen at the connection point of the new feeder.

### 7.8.3. Results

Tables 7.3 to 7.6 show the reliability indices for the new bay with years and simulations of  $n = 10\,000$  realizations. Figures 7.6 and 7.7 show and for the study cases.

Required time for simulating these cases was in average 24 hours using common desktop computers (Intel Core 2 processor, 2.4 and 2.66 GHz, 2 to 3 GB of RAM).

Year	$\lambda_o$	$\lambda_f$	$\lambda_m$	$\lambda_{fo}$	$\lambda_{bu}$
1	7.2314	4.0330	3.0124	0.0892	0.0968
2	15.2035	11.7570	3.0124	0.1208	0.3133
3	15.6872	12.2442	3.0124	0.1244	0.3062

**Table 7.3.** Expected event rates in [events/year] for PZ5 – Case 1

Year	$U_o$	$U_f$	$U_m$	$U_{fo}$	$U_{bu}$
1	0.5313	0.1111	0.4110	0.0075	0.0018
2	0.7532	0.3262	0.4109	0.0097	0.0063
3	0.7660	0.3380	0.4118	0.0103	0.0060

**Table 7.4.** Expected unavailabilities in [%] for PZ5 – Case 1

Year	$\lambda_o$	$\lambda_f$	$\lambda_m$	$\lambda_{fo}$	$\lambda_{bu}$
1-2-3	12.1913	8.8914	3.0124	0.0936	0.1939

**Table 7.5.** Expected event rates in [events/year] for PZ5 – Case 2

Year	$U_o$	$U_f$	$U_m$	$U_{fo}$	$U_{bu}$
1-2-3	0.6652	0.2429	0.4111	0.0077	0.0036

**Table 7.6.** Expected unavailabilities in [%] for PZ5 – Case 2

#### 7.8.4. Analysis of the Results

Results of case 1 show that although PZ5 has new substation equipment, its reliability indices increase over time due to the presence of aged substation equipment in the other bays.

On the other hand, for most term of the study, the reliability indices obtained in case 2 are lower than those obtained in case 1.

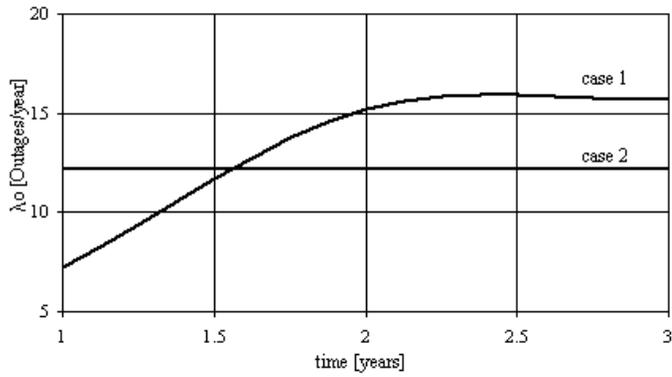


Figure 7.6. Expected operational outage rate for the new bay

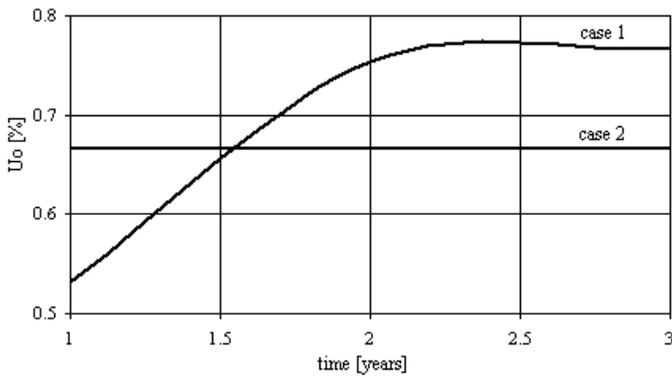


Figure 7.7. Expected operational unavailability for the new bay

This shows the error that exists when the reliability of a system with aged components is assessed assuming that all component failure processes are stationary.

### 7.9. Conclusions

Stochastic Point Processes and the Monte Carlo simulation allow implementing a method for reliability assessment of substations that greatly improves the modeling detail of these kinds of studies.

It can manage time-varying rates and allows a detailed representation of the protective systems operating sequence and the effect of their failures. Additionally, it does not require an exhaustive list of operating states as other methods do. However, this improvement in modeling detail has a price—the long computational time required by the simulation.



## CHAPTER 8

# MODELING OF FAILURES TO OPERATE OF PROTECTIVE SYSTEMS FOR RELIABILITY STUDIES AT THE POWER SYSTEM LEVEL

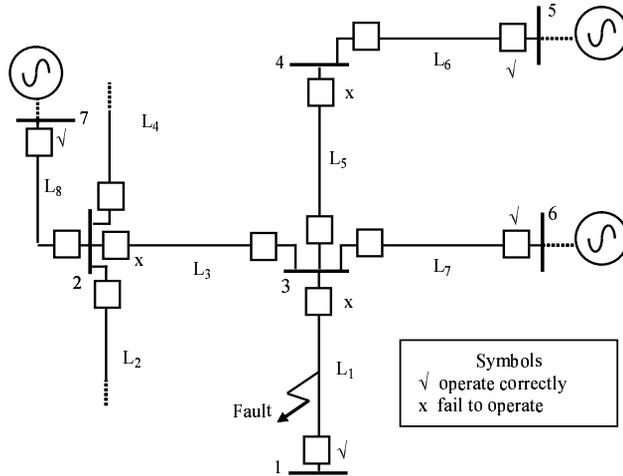
The aim of this chapter is to present the method for obtaining a condensed model of protective system failures to operate that will be incorporated in the assessment of the power system. The facts that justify this kind of representation are also discussed.

The content of this chapter is taken from the paper “Modeling of Protective System Failures to Operate for Reliability Studies at the Power System Level Using Stochastic Point Processes,” by Zapata, Kirschen, Torres and Ríos [30].

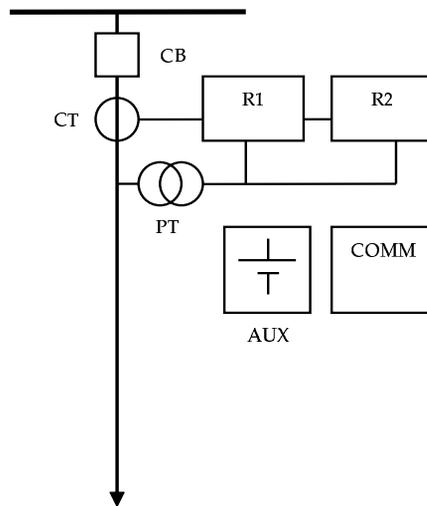
### **8.1. Power System Reliability Assessments Considering PS Failures to Operate**

To illustrate how a power system reliability study that considers PS failures to operate works, let us consider the PS at each trans-

mission line terminal in the power system shown in Figure 8.1, which has the scheme shown in Figure 8.2.



**Figure 8.1.** Analysis on the effect of protective system failures to open



**Figure 8.2.** Protective system at a terminal of a transmission line

For a fault on line  $L_1$ , it is necessary to analyze if the PS at terminals 1 and 3 operate correctly, i.e., if they open. If the PS at terminal 3 FTO, it is then necessary to analyze if the PS at terminals 2, 4 and 6 of transmission lines  $L_3$ ,  $L_5$  and  $L_7$ , respectively, operate correctly, and so on. Thus, for this kind of study, it is necessary to represent the PS associated to each PZ. There are two main approaches for this task:

1. To incorporate the reliability models of the PSC associated to each PS.
2. To condense the effect of all PSC associated to a given PS into the reliability models of the circuit breakers associated to the PZ. This can be done because all PS failures are reflected on the PZ circuit breakers, no matter the PSC that caused them.

The first approach is the less popular one because it demands more computer processing capacity (RAM) and computing time, due to the huge number of reliability models that have to be evaluated. To illustrate this, let us consider a reliability study of the power system shown in Figure 8.1. If it only considers failures on the transmission lines, it will require 16 failure models (8 for permanent failures + 8 for temporary failures). If it also considers FTO, it is necessary to incorporate 224 additional failure models in the first approach, but only 16 in the second one. These numbers are obtained in the following way:

- $224 = (8 \text{ transmission lines}) * (2 \text{ terminals per transmission line}) * (7 \text{ PSC per PS}) * (2 \text{ failure modes per PSC (FTO + KND)})$ ;
- $16 = (8 \text{ transmission lines}) * (2 \text{ terminals per transmission line}) * (1 \text{ circuit breaker per terminal})$ .

A common practice in the first approach is to restrict the analysis to the PSC considered “most important,” viz., circuit breakers, current transformers and relays; however, this also decreases the level of detail of the reliability study.

Regarding the second approach, the condensed model at each circuit breaker is expressed in the form of a probability for each type of PS failure to operate. These probabilities are computed before and out of the power system reliability study, using operating data or by means of a reliability assessment of the protective scheme.

The second method of obtaining the probabilities of the condensed model is applied in those situations where the aim is to evaluate the impact of incorporating PSC with different levels of reliability and of considering diverse protective schemes.

## 8.2. Problem Statement

Several modeling tools have been applied to obtain a condensed model of PS failures to operate: homogeneous Markov chains, event trees, fault trees, cut sets and reliability blocks. All of them work under the assumption that PSC failure and repair processes are stationary; this is expressed by means of constant failure and repair rates, constant probabilities of failure or constant availabilities.

Although stationarity has long been a common assumption in power system reliability, its relevance should be carefully re-examined, due to the growing importance of factors such as aging and improvement/decrease in maintenance resources. If stationarity is no longer a valid assumption, the application of the mathematical methods mentioned above is no longer valid. For this reason, this author has proposed a method based on SPP model-

ing that can manage constant or time-varying rates. It is applied here for obtaining the condensed model of a PS.

### 8.3. Proposed Method

Using the assessment procedure described in Chapter 6, the probabilities of the condensed model are computed in the following way:

$$P[FTO]_{lk} = \frac{FTO_k}{CTO_k}; \tag{8.1}$$

$$P[FTC]_{lk} = \frac{FTC_k}{CTC_k}. \tag{8.2}$$

### 8.4. Example

Consider the test system presented in Chapter 6 and the same study cases. Tables 8.1 to 8.3 show the results for  $T = 3$  years and  $n = 10000$  realizations. Figure 8.3 shows the probability of FTO at circuit breaker 13 for the three study cases.

Simulations lasted 0.47 hours, 5.34 hours and 8.81 hours for cases 1, 2 and 3, respectively. The post-processing of the simulation outputs to compute the probabilities of the condensed model lasted less than fifteen minutes in all study cases.

TIME [Years]	$P[FTO]_{l_1}$	$P[FTO]_{l_2}$	$P[FTO]_{l_3}$	$P[FTO]_{l_{1:2}}$	$P[FTO]_{l_{1:3}}$	$P[FTO]_{l_{2:3}}$	$P[FTO]_{l_{1:2:3}}$
1.0–3.0	0.0484	0.0462	0.0328	0.0000	0.0000	0.0000	0.0080

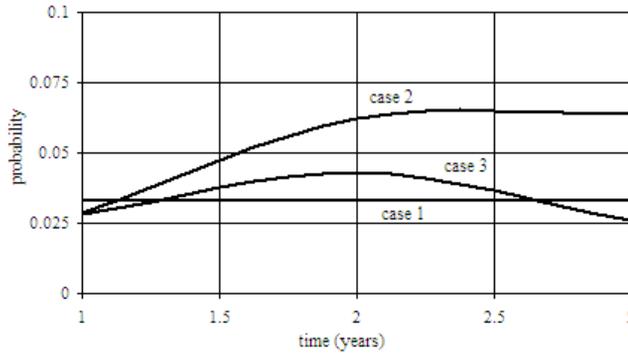
**Table 8.1.** FTO probability of the protective system – Case 1

TIME [Years]	$P[FTO]_{11}$	$P[FTO]_{12}$	$P[FTO]_{13}$	$P[FTO]_{11-12}$	$P[FTO]_{11-13}$	$P[FTO]_{12-13}$	$P[FTO]_{11-12-13}$
1.0	0.0420	0.0414	0.0285	0.0000	0.0000	0.0000	0.0071
2.0	0.1071	0.0937	0.0625	0.0000	0.0000	0.0000	0.0191
3.0	0.1199	0.1070	0.0637	0.0000	0.0000	0.0000	0.0182

**Table 8.2.** FTO probability of the protective system – Case 2

TIME [Years]	$P[FTO]_{11}$	$P[FTO]_{12}$	$P[FTO]_{13}$	$P[FTO]_{11-12}$	$P[FTO]_{11-13}$	$P[FTO]_{12-13}$	$P[FTO]_{11-12-13}$
1.0	0.0419	0.0413	0.0284	0.0000	0.0000	0.0000	0.0071
2.0	0.0698	0.0620	0.0431	0.0000	0.0000	0.0000	0.0110
3.0	0.0410	0.0402	0.0263	0.0000	0.0000	0.0000	0.0050

**Table 8.3.** FTO probability of the protective system – Case 3



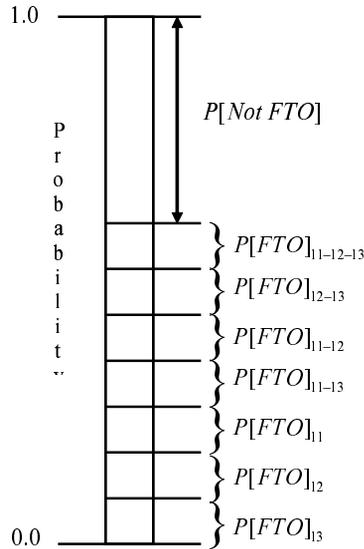
**Figure 8.3.** FTO probability at circuit breaker 13

### 8.5. How to Use this Model

To illustrate how the PS condensed model is applied, let us consider the sketch shown in Figure 8.4.

For a given PS failure mode, for example FTO, the probabilities of occurrence of a FTO at each circuit breaker at a given time are a part of the total probability of the sample space. The sample space also includes the event of not occurrence of FTO.

To sample this model in the reliability assessment at the power system level, a  $U$  is generated each time a failure affects the PZ; the value of  $U$  defines the event that occurs. For example, using data



**Figure 8.4.** Sample space for failures to open at circuit breakers

of Table 8.3, for case 1, if in the first year a failure affects the power transformer and  $U = 0.00251$ . This means circuit breaker 11 fails to open.

### 8.6. Conclusion

A new method for representing failures to operate of protective systems in reliability studies at the power system level is presented in this chapter. Like other methods that have been applied for this task, it condenses at the circuit breakers of the protection zone the effect of protective component failures, protective scheme configuration and maintenance strategies, but unlike them it supports the consideration of time-varying failure and repair rates.



## CHAPTER 9

# THE LOSS OF COMPONENT SCENARIO METHOD TO ANALYZE THE VULNERABILITY OF A COMPOSITE POWER SYSTEM

This chapter presents the method for assessing the vulnerability of a composite power system. It gathers all the concepts and methods presented in previous chapters.

Most of the content of this chapter is taken from the paper “A Method for Studying Loss of Component Scenarios in a Power System Using Stochastic Point Processes,” by Zapata, Torres, Kirschen and Ríos [27].

### 9.1. Definition of Loss of Component Scenario

A loss of component scenario (LCS) is the situation where a power system with  $n$  components has  $x$  components out of service due to planned or unplanned events. It is denoted as a  $n - x$  LCS and it is called an LCS of order  $x$ .

A high-order LCS is defined here as the situation where  $x > 2$ .



systems and protective relays). Thus, component failure models include failures of both types. If each component has two states, “available” and “unavailable,” the state space of operating states of a system with  $n$  components has a dimension of 2.

The term *operating state* refers here to an LCS and does not qualify the system operating condition in terms such as “Normal,” “Alert,” or “Emergency,” as used in the context of power system security.

Because every power system has many components, the dimension of the space of operating states is huge; hence, every study on the occurrence of LCS is a cumbersome task, no matter the assessment method applied.

## 9.2. Objective of an LCS Study

The aim of this study is to measure the occurrence of LCS by means of the following indices:

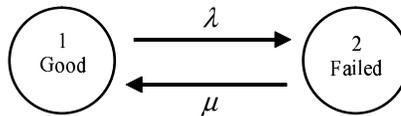
- the probability of occurrence  $P_{n-x}$ ;
- the expected frequency  $F_{n-x}$ ; and
- the mean duration  $D_{n-x}$ .

Although this objective may appear similar to the one of the reliability assessment technique called “state enumeration,” the difference is that in the present study the indices are determined globally for the set of operating states that belong to a given LCS order and not for each of them separately.

For example, considering the system shown in Figure 9.1, LCS indices are calculated for the  $n - 2$  LCS order and not for each one of the 22 operating states that belong to this order.

### 9.3. Traditional Modeling for Reliability Studies

For simplicity, in the following discussion, only the basic component reliability model of two operating states shown in Figure 9.2 is considered. However, the proposed method extends to models with any number of states.



**Figure 9.2.** Two-state component reliability model

This model is defined by means of a failure rate  $\lambda$  and a repair rate  $\mu$ , which can be estimated from operating records that cover a period  $T$  in the following way:

$$\lambda = \frac{1}{\overline{tff}} = \frac{n_f}{T - \sum_{j=1}^{n_r} \overline{ttr}_i}, \quad (9.1)$$

$$\mu = \frac{1}{\overline{ttr}} = \frac{n_r}{T - \sum_{j=1}^{n_f} \overline{tff}_i}, \quad (9.2)$$

where  $\overline{tff}$ ,  $n_f$ ,  $\overline{ttr}$  and  $n_r$  denote respectively the time to failure, the number of failures, the time to repair and the number of repairs. Over-lined symbols such as  $\overline{x}$  denote the statistical mean of the variable.

Two important component reliability indices are the availability ( $A$ ) and the unavailability ( $U = \overline{A}$ ), which can be estimated

from operating records in the following way:

$$A = P_1(\infty) = \frac{\mu}{\lambda + \mu} = \frac{1}{T} \sum_{j=1}^{n_f} ttf_j, \quad (9.3)$$

$$U = P_2(\infty) = \frac{\lambda}{\lambda + \mu} = \frac{1}{T} \sum_{j=1}^{n_r} ttr_j, \quad (9.4)$$

where  $P_i(\infty)$  denotes the probability of finding state  $i$  in the long run. Constant failure and repair rates imply that the failure and repair processes are stationary and hence, in the long run,  $A$  and  $U$  tend towards the estimates (9.3) and (9.4) independently of the probability distribution of  $ttf$  and  $ttr$ .

If the distributions of  $ttf$  and  $ttr$  of all components are exponential, the LCS study can be solved using the continuous Markov chain method. Figure 9.3 shows a Markov chain representation for a system with  $n$  components.

The mathematical formulation of this method for a system of  $n$  components is the set of ordinary differential equations:

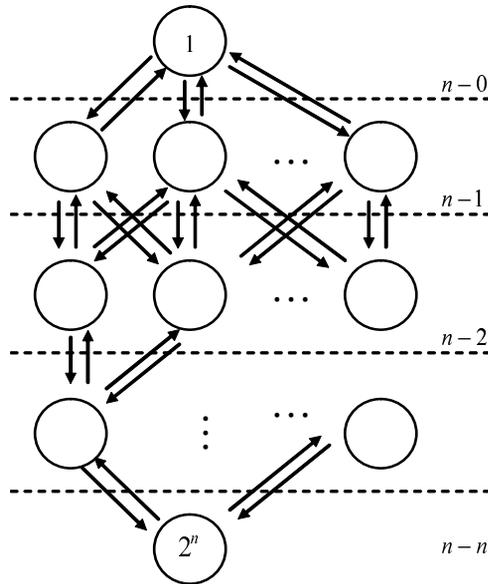
$$[\dot{P}]^t = [M]^t [P]^t, \quad (9.5)$$

where  $[P]$  is a vector with the probabilities of the  $n * n$  states, and  $[M]$  is the matrix of transition rates between states.

If  $k$  operating states are LCS of order  $x$  the probability of occurrence of LCS of this order is:

$$P_{n-x} = \sum_{j=1}^k P_j(\infty). \quad (9.6)$$

The expected frequency and the mean duration of LCS of order



**Figure 9.3.** Markov chain diagram for the state space of system operating states

$x$  are computed as:

$$F_{n-x} = P_{n-x} * \sum h, \tag{9.7}$$

$$D_{n-x} = \frac{1}{\sum h}, \tag{9.8}$$

where  $\sum h$  is the sum of the rates of all the transitions leaving states belonging to an LCS of a particular order, i.e., the transitions that cross one of the dashed lines in Figure 9.3.

If the failure and repair processes of all components are independent, it is not necessary to solve (9.5) first to apply (9.6). Each term of (9.6) can be obtained by applying the formula of

simultaneous occurrence of independent events ( $E_i$ ):

$$P[E_1 E_2 \dots E_N] = \prod_{j=1}^N P[E_j]. \quad (9.9)$$

where  $E_i$  denotes the operating state ( $A_i$  or  $\bar{A}_i$ ) of component  $i$ .

If all component failure and repair processes are stationary and independent, (9.9) is valid no matter the probability distributions used to represent these processes.

It is important to observe that (9.9) does not apply for the case where common failure modes exist and its extension to include them is very complicated. In this case, common failure modes have to be included in  $M$ , which can then be solved by applying (9.5). It is also important to point out that before the application of (9.5) or (9.9), a list of all possible operating states has to be elaborated and this is not an easy task.

## 9.4. What Do the Assumptions of Traditional Modeling Imply?

### 9.4.1. Stationary Failure and Repair Processes

Stationarity of failure and repair processes has been a common assumption in the field of power system reliability. However, the validity of this assumption must be examined carefully, especially for the failures. Because of component aging and lack of preventive maintenance—two conditions currently present in many power systems all over the world—component failure rates may not be constant but rather increasing functions of time.

On the other hand, a constant repair rate means that the performance of repair teams is not affected by internal or external factors. However, in real life, crew performance is affected by ex-

ternal factors such as weather and traffic, as well as by internal factors such as available tools and personnel skills.

Non-constant failure and repair rates also mean that the study of LCS cannot be done by applying (9.3) to (9.9). One way of overcoming this problem is to introduce time-varying rates into  $M$  to solve (9.5), i.e., by making it into a non-homogeneous Markov chain process. However, this kind of process has problems with the adjustment of the operating times and with tractability for some types of time-varying rates. Even if the limitations of the non-homogeneous Markov chain process were considered unimportant, this kind of modeling has another drawback that is discussed next.

#### **9.4.2. Independent Component Repair Processes**

It has also been a common practice in the field of power system reliability to include repairs as part of component models; this approach means that:

1. The repair process of each component is independent of the repair processes of other components.
2. Repair resources are unlimited because every time a component fails it is immediately repaired. This is equivalent to assuming that each component has a dedicated repair team.

Obviously, these assumptions are unrealistic. For maintenance purposes, a power system is typically split into maintenance zones which are assigned to repair teams. Repair resources are limited because crews have to fix all failed components located in their zone. Hence, some failures have to wait until the ones that occurred first are repaired. Thus, repairs of component located in a given maintenance zone are not independent.

### 9.4.3. Proposal

To improve the modeling flexibility of the LCS study, it is proposed to represent the component failure processes using an SPP and the repair process of each maintenance zone of the power system using concepts from queuing theory.

## 9.5. Proposed Method

### 9.5.1. Failure Process Modeling

#### 9.5.1.1. *Components with Two Operating States*

The failure process of a two-state component is represented by means of an SPP model.

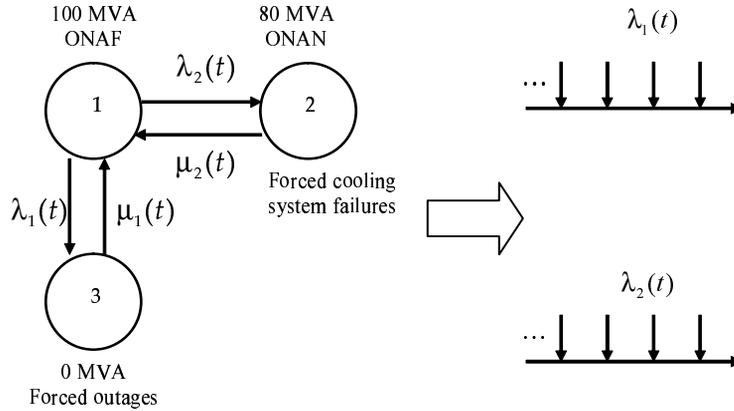
#### 9.5.1.2. *Components with More than Two Operating States*

Some people believe components with more than two operating states cannot be represented by SPP. This is not true. In this case, the component is represented by one SPP model for each operating state (rated, derated and failed states).

For example, consider the Markov chain model for a power transformer with a forced cooling system shown to the left in Figure 9.4.  $\lambda_1(t)$  is the forced failure rate and  $\lambda_2(t)$  the failure rate of the forced cooling system. In this case, the failure process of the power transformer is represented by two independent SPP as shown to the right in Figure 9.4.

#### 9.5.1.3. *Common Mode Failures*

Each common failure mode is represented by means of an SPP model. This means that common mode failures have to be distinguished from other types of component failures. Because the



**Figure 9.4.** SPP modeling for a component with three operating states

definition of SPP imposes that only one event can occur at any instant, common mode failures are considered point events with which a given number of component disconnections is associated.

*9.5.1.4. Longitudinal Components*

A longitudinal component can be located in several maintenance zones. To properly manage this, two modeling approaches are considered:

- To split the longitudinal component in as many sub-components as maintenance zones it belongs. This means that failure data can be classified by maintenance zone in order to fit an SPP failure model for each subcomponent. The failure of any subcomponent produces a common mode outage on the other ones.
- To use a single SPP failure model and to associate to it a distribution model  $F_L(L)$  for the distance  $L$  from a component terminal to the failure point. Every time a component failure

is generated using the SPP failure model of the component, a distance to failure  $L$  is generated using  $F_L(L)$ ; thus,  $L$  gives information about the maintenance zone in which the failure is located.

### 9.5.2. Repair Process Modeling

The alternatives for defining the repair service model are:

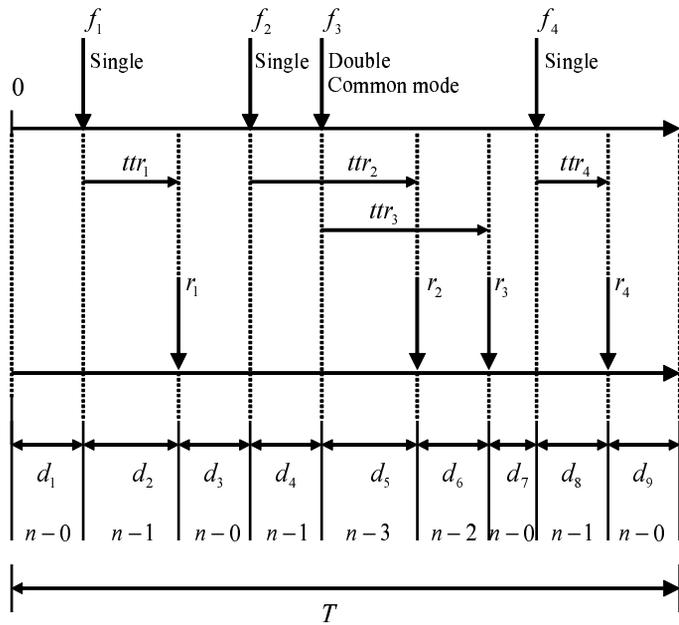
- To use an RP or an NHPP.
- To generate a repair time in accordance with the type of equipment to be repaired. In this case, the repair service SPP is non-homogeneous, in the sense that repair times sequences are not generated using a single distribution.

### 9.5.3. Algorithm of the Proposed Method

A sequential Monte Carlo simulation is used to generate the sequences of failures and repairs in each maintenance zone of the power system. The steps of this procedure are:

1. Generate for the period of study  $T$  of one or more years the failure processes of all the components, i.e., generate for each component and common failure mode a sequence of times to failure in accordance with its SPP model.
2. For each maintenance zone of the power system, superpose the failure processes of all components located in that zone.
3. For each maintenance zone of the power system, generate the repair process using the SPP repair service model, i.e., for each failure that occurs in a given zone, generate a time to repair in accordance with the SPP repair service model for that zone.

4. Failures and repairs define sub-intervals of different LCS order, as shown in Figure 9.5. Each failure  $f_i$  has an associated repair  $r_i$  that lasts  $ttr_i$ , but some repairs are delayed due to congestions in the repair system, i.e., some failures have to wait to be repaired until the repairs of other components that failed before are finished.



**Figure 9.5.** Sequences of failures and repairs for a sample

5. For each LCS order, count the number of subintervals  $N_{(n-x)}$  and compute the sum  $d_{(n-x)}$  of these intervals.
6. Repeat steps 1 to 4 for  $N$  samples.

7. Compute for each sub-period  $k$  (month, semester, year, etc.) of  $T$  the LCS indices in the following way:

$$P_{n-x} = \frac{\sum_{j=1}^N d_{(n-x)_j}}{N * T}; \quad (9.10)$$

$$D_{n-x} = \frac{\sum_{j=1}^N d_{(n-x)_j}}{N}; \quad (9.11)$$

$$F_{n-x} = \frac{\sum_{j=1}^N N_{(n-x)_j}}{N * T}. \quad (9.12)$$

Contrary to (9.5) and (9.9), the proposed method does not require a table of possible LCS.

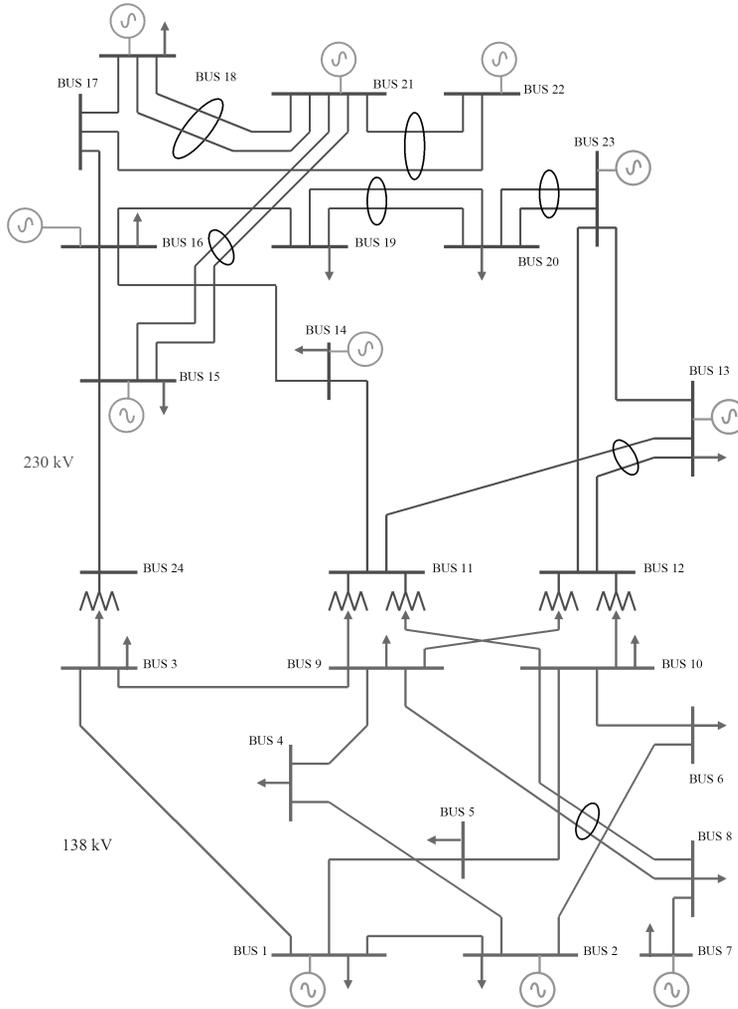
When using time-varying rates, (9.10)-(9.12) should not be computed for a single sub-period equal to  $T$ , because variation is lost; also, all SPP models have to be synchronized to a common time reference.

### 9.6. Examples

The proposed method is applied to a study of LCS in the transmission zone of the one-area IEEE Reliability Test System (RTS) [7] shown in Figure 9.6.

As pointed out by Billinton and Allan [4]:

*“In many cases, severity associated with a contingency event is inversely related to the frequency and the probability of its occurrence. In other words, as the*



**Figure 9.6.** The Single Area IEEE RTS

*number of components involved in a simultaneous outage increases, both the probability and the frequency of the contingency decrease.”*

The analysis of results presented here thus focuses more on the relative change in LCS indices than in their absolute magnitudes.

### 9.6.1. Example 1: Constant Rates and Diverse Repair Logistics

This example considers the following cases:

Case 1: Each component has a repair team dedicated to it. Results are the same as those obtained when applying (9.5)-(9.9). Failure and repair processes of all components are modeled as HPP using RTS data.

Case 2: Three maintenance zones are considered; one for the 138 kV transmission lines, one for the power transformers and one for the 230 kV transmission lines. The repair service process in each maintenance zone follows the RTS repair data.

Case 3: Similar to Case 2, but now three operating states are considered for the power transformers, as shown in Figure 9.8. The nominal capacity (ONAF) state has failure rate equivalent to 80% of the value given for the RTS and a mean repair time of 768 hours (RTS data). The derated capacity (ONAN) state has failure rate equivalent to 20% the value given for the RTS and an assumed repair time of 168 hours (one week).

Tables 9.1 to 9.3 show the results for  $T = 1$  year and simulations for 15 000 realizations. Relative changes in results ( $\Delta\%$ ) for case 2 are computed with reference to case 1, and for case 3 are computed with reference to case 2.

These results show that:

- For the system under study, LCS of order up to 4 are expected to occur.

LOCS	Case 1	Case 2		Case 3	
			$\Delta\%$		$\Delta\%$
$n-0$	97.9410	97.9157	-0.03	97.9724	0.06
$n-1$	2.4342	2.4357	0.06	2.3084	-5.23
$n-2$	0.0451	0.0570	26.39	0.0578	1.40
$n-3$	7.6970E-04	0.0011	42.91	0.0011	0.00
$n-4$	1.7158E-06	3.6516E-06	112.82	3.8405E-06	5.17

**Table 9.1.** Example 1 – Probability of occurrence of LCS [%]

LOCS	Case 1	Case 2		Case 3	
			$\Delta\%$		$\Delta\%$
$n-0$	13.8680	13.8687	0.01	13.9001	0.23
$n-1$	12.9643	12.9625	-0.01	12.9705	0.06
$n-2$	0.4726	0.4727	0.02	0.4653	-1.57
$n-3$	0.0111	0.0112	0.90	0.0105	-6.25
$n-4$	6.6667E-05	2.0000E-04	200.00	1.3333E-04	-33.34

**Table 9.2.** Example 1 – Expected frequency of LCS [events/year]

LOCS	Case 1	Case 2		Case 3	
			$\Delta\%$		$\Delta\%$
$n-0$	618.6640	618.4745	-0.03	617.4345	-0.17
$n-1$	16.4477	16.4602	0.08	15.5905	-5.28
$n-2$	8.3644	10.5716	26.39	10.8890	3.00
$n-3$	6.0927	8.2318	35.11	9.3679	13.80
$n-4$	2.2545	1.5994	-29.06	2.5232	57.76

**Table 9.3.** Example 1 – Mean duration of LCS [hours/event]

- When considering limited repair resources (case 2)  $P_{n-x}$  of LCS of orders 2, 3 and 4 have important increments.
- The effect of three-state modeling for power transformers is observed as decrements in  $F_{n-x}$  and increments in  $D_{n-x}$  for LCS of orders 2, 3 and 4.

Required time for simulating these cases was in average 2.5 hours using common desktop computers (Intel Core 2 processor, 2.4 and 2.66 GHz, 2 to 3 GB of RAM).

**9.6.2. Example 2: Increasing Failures Rates – Constant Repair Rates**

This example considers three maintenance zones with the same repair models as in case 2 of example 1, but the failure rates are now considered increasing functions of time, in order to represent a situation of aging and lack of maintenance. Failure processes are modeled using a PLP . The scale parameter  $\lambda$  is defined as the permanent failure rate given as data for the RTS and a shape parameter  $\beta = 1.5$ .

Tables 9.4 to 9.6 show results for year, two sub-periods per year and simulations of 12 500 realizations. Relative changes ( $\Delta\%$ ) in the indices are shown in Figures 9.7 to 9.9. These values are computed with reference to the index value in the first sub-period where it is non-zero. In Figure 9.7 results for LCS of order zero cannot be seen due to its low magnitudes. In Figure 9.9 only changes in mean duration for LCS of order zero, one and two are presented; results for LCS of orders 3 and 4 are not presented because they show high oscillation.

	<i>n-0</i>	<i>n-1</i>	<i>n-2</i>	<i>n-3</i>	<i>n-4</i>
0.25	98.9949	1.0601	0.0217	8.2634E-04	0
0.50	98.0619	2.1849	0.0479	0.0015	0
0.75	97.4827	2.9119	0.0707	0.0022	6.7696E-05
1.00	97.1371	3.3766	0.0852	0.0021	3.8474E-05
1.25	96.7827	3.7999	0.1159	0.0031	1.3545E-04
1.50	96.4507	4.2922	0.1468	0.0035	2.0637E-04
1.75	96.1390	4.6838	0.1505	0.0085	2.5114E-04
2.00	95.9362	5.0563	0.1823	0.0051	1.4517E-04
2.25	95.7195	5.2416	0.1919	0.0084	2.4901E-04
2.50	95.4847	5.5996	0.2157	0.0067	1.4172E-04
2.75	95.2456	5.8907	0.2363	0.0075	2.5307E-04
3.00	95.0697	6.2328	0.2538	0.0086	4.6117E-04

**Table 9.4.** Example 2 – Probability of occurrence of LCS [%]

	<i>n-0</i>	<i>n-1</i>	<i>n-2</i>	<i>n-3</i>	<i>n-4</i>
0.25	10.4064	6.4323	0.1952	0.00420	0
0.50	15.7613	11.9642	0.4163	0.0109	0
0.75	19.1667	15.4944	0.6243	0.0170	3.2000E-04
1.00	21.6378	18.1136	0.7789	0.0211	6.4000E-04
1.25	24.0669	20.6992	0.9715	0.0317	0.0013
1.50	26.0259	22.7818	1.1613	0.0381	9.6000E-04
1.75	28.1152	25.0173	1.3354	0.0406	0.0022
2.00	29.8410	26.8659	1.5104	0.0688	0.0013
2.25	31.2595	28.3568	1.6176	0.0742	0.0026
2.50	32.7146	29.9389	1.8112	0.0765	0.0019
2.75	34.3069	31.6848	1.9613	0.0896	0.0032
3.00	35.5818	33.1408	2.1238	0.0925	0.0042

Table 9.5. Example 2 – Expected frequency of LCS [events/year]

	<i>n-0</i>	<i>n-1</i>	<i>n-2</i>	<i>n-3</i>	<i>n-4</i>
0.25	833.3288	14.4366	9.7471	17.4008	0
0.50	545.0207	15.9973	10.0751	11.8584	0
0.75	445.5372	16.4626	9.9152	11.2794	18.5319
1.00	393.2573	16.3299	9.5806	8.8006	5.2661
1.25	352.2750	16.0812	10.4486	8.5726	9.2699
1.50	324.6411	16.5043	11.0700	8.0992	18.8316
1.75	299.5455	16.4008	9.8747	18.4118	9.8214
2.00	281.6266	16.4869	10.5738	6.4494	9.9349
2.25	268.2393	16.1923	10.3940	9.8709	8.5207
2.50	255.6801	16.3841	10.4331	7.7128	6.4661
2.75	243.2023	16.2861	10.5520	7.2913	6.9277
3.00	234.0554	16.4750	10.4688	8.1406	9.7112

Table 9.6. Example 2 – Mean duration of LCS [hours/event]

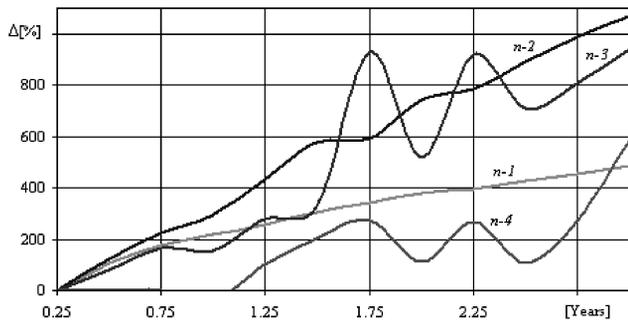


Figure 9.7. Change on LCS probability of occurrence – Example 2

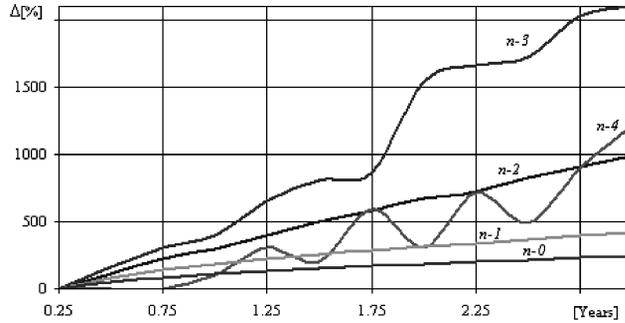


Figure 9.8. Change on LCS expected frequency – Example 2

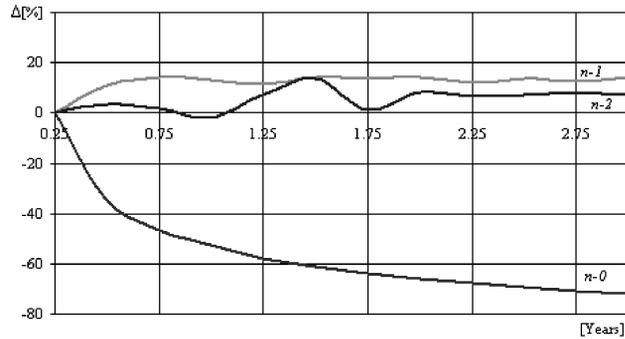


Figure 9.9. Change on LCS mean duration – Example 2

These results show that:

- As time passes,  $P_{n-x}$  and  $F_{n-x}$  for LCS of orders 1 to 4 are increasing functions of time. The increments in these indices are huge.
- For the  $n - 0$  operating state,  $D_{n-x}$  is the index that better reflects the effect of increasing failure rates and constant repair resources. It is a decreasing function of time.

Required time for simulating this case was in average 6.0 hours using common desktop computers (Intel Core 2 processor, 2.4 and 2.66 GHz, 2 to 3 GB of RAM).

## 9.7. Conclusions

Due to the huge dimensions of the space of operating states of a power system, a study of LCS is always a cumbersome task, no matter what method is applied. The proposed method is a very flexible modeling alternative but it requires significant computer resources.

The presence of aging and other factors that produce increasing component failure rates increases the risk of occurrence of high-order loss of component scenarios. On the other hand, maintenance strategies can decrease this risk. For this reason, it is necessary an assessment method as the one proposed here, which can manage time-varying rates.

## CHAPTER 10

### MAIN CONCLUSION

Vulnerability assessments of composite power systems have to include all those factors that can produce increasing component failure rates, such as aging and lack of resources for maintenance, because they dramatically increase the risk that high-order loss of component scenarios occur.

This also means that the vulnerability assessments must be performed using methods than can manage time-varying rates. To address this need, a method based on stochastic point process modeling and a sequential Monte Carlo simulation is proposed in this work.



## REFERENCES

- [1] H. Ascher and H. Feingold. *Repairable Systems Reliability: Modeling, Inference, Misconceptions and Their Causes*. Marcel Dekker, 1984.
- [2] H.E. Ascher and C.K. Hansen. Spurious exponentially observed when incorrectly fitting a distribution to nonstationary data. *IEEE Transactions on Reliability*, 47(4), Dec. 1998.
- [3] N. Balijepalli, S. Venkata Subrahmanyam, and R.D. Christie. Modeling and analysis of distribution reliability indices. *IEEE Transactions on Power Delivery*, 19(4), Oct. 2004.
- [4] R. Billinton and R.N. Allan. *Reliability Assessment of Large Electric Power Systems*. Kluwer, 1988.
- [5] C.A. Dortolina, J.J. Porta, and R. Nadira. An approach for explicitly modeling the protective relaying system in substation reliability studies. *IEEE Transactions on Power Systems*, 6(4), 1991.
- [6] T.F. Hassett, D.L. Dietrich, and F. Szidarovszky. Time-varying failure rates in the availability & reliability analysis of repairable systems. *IEEE Transactions on Reliability*, 44(1), Mar. 1995.

- [7] IEEE Task Force. The IEEE reliability test system - 1996. *IEEE Transactions on Power Systems*, 14(3), Aug. 1999.
- [8] IEEE Task Force. Vulnerability assessment for cascading failures in electric power systems. *IEEE PES General Meeting*, 2009.
- [9] B. Klefsjo and U. Kumar. Goodness-of-fit tests for the power law process based on the TTT plot. *IEEE Transactions on Reliability*, 41(4), Dec. 1992.
- [10] V.I. Kogan and R.J. Gursky. Transmission towers inventory. *IEEE Transactions on Power Delivery*, 11(4), Oct. 1996.
- [11] V.I. Kogan and T.L. Jones. An explanation for the decline in URD cable failures and associated nonhomogeneous poisson process. *IEEE Transactions on Power Delivery*, 9(1), Jan. 1994.
- [12] M.T. Schilling, J.C.G. Praca, J.F. Queiroz, C. Singh, and H. Ascher. Detection of ageing in the reliability analysis of thermal generators. *IEEE Transactions on Power Systems*, 3(2), 1988.
- [13] R.H. Stillman. Modeling failure data of overhead distribution systems. *IEEE Transactions on Power Delivery*, 15(4), Oct. 2000.
- [14] R.H. Stillman. Power line maintenance with minimal repair and replacement. *IEEE Annual Reliability and Maintainability Symposium*, 2003.
- [15] W.A. Thomson. *Point Processes Models with Applications to Safety and Reliability*. Chapman and Hall, 1988.

- [16] J.F. Valencia. Confiabilidad de sistemas de protección de transformadores de potencia del sistema CHEC [Reliability of protective systems of power transformers of CHEC]. Graduation Project, Universidad de los Andes, Bogotá, 2009. (In Spanish).
- [17] C.J. Zapata. Análisis probabilístico y simulación [Probabilistic analysis and simulation]. Universidad Tecnológica de Pereira, 2010. (In Spanish).
- [18] C.J. Zapata, A. Alzate, and M. Ríos. Reliability assessment of substations using stochastic point processes and Monte Carlo simulation. *IEEE PES General Meeting*, 2009.
- [19] C.J. Zapata, C.M. Arbeláez, and C.A. Pulgarín. Índices de confiabilidad de equipos de subestaciones de distribución [Reliability indices of distribution substation equipment]. *Mundo eléctrico*, 65, 2006. (In Spanish).
- [20] C.J. Zapata, D.Y. Cataño, and H.F. Suárez. Índices de confiabilidad de transformadores de distribución [Reliability indices of distribution transformers]. *Mundo eléctrico*, 57, 2004. (In Spanish).
- [21] C.J. Zapata and O.J. Hernández. Índices de confiabilidad de transformadores de potencia de subestación de distribución [Reliability indices of distribution substation power transformers]. *Mundo eléctrico*, 71, 2008. (In Spanish).
- [22] C.J. Zapata, M.E. Leyes, and M.Y. Patiño. Índices de confiabilidad de circuitos primarios de distribución [Reliability indices of distribution feeders]. *Mundo eléctrico*, 69, 2007. (In Spanish).

- [23] C.J. Zapata and P.A. Montealegre. Problemas técnicos en la calidad del servicio de electricidad [Technical problems on the quality of electricity service]. *Mundo eléctrico*, 68, 2007. (In Spanish).
- [24] C.J. Zapata and J.A. Ramírez. Índices de confiabilidad de cortacircuitos en distribución [Reliability indices of distribution cut-outs]. *Mundo eléctrico*, 66, 2007. (In Spanish).
- [25] C.J. Zapata, S.C. Silva, O.L. Burbano, and J.A. Hernández. Repair models of power distribution components. *IEEE Transmission & Distribution Latin America Conference & Exhibition*, 2008.
- [26] C.J. Zapata, S.C. Silva, H.I. González, O.L. Burbano, and J.A. Hernández. Modeling the repair process of a power distribution system. *IEEE Transmission & Distribution Latin America Conference & Exhibition*, 2008.
- [27] C.J. Zapata, A. Torres, D.S. Kirschen, and M. Ríos. A method for studying loss of component scenarios in a power system using stochastic point processes. *IEEE PES General Meeting*, 2009.
- [28] C.J. Zapata, A. Torres, D.S. Kirschen, and M. Ríos. Reliability assessment of protective schemes considering time varying rates. *International Review of Electrical Engineering*, 4(6), 2009.
- [29] C.J. Zapata, A. Torres, D.S. Kirschen, and M. Ríos. Some misconceptions about the modeling of repairable components. *IEEE PES General Meeting*, 2009.
- [30] C.J. Zapata, A. Torres, D.S. Kirschen, and M. Ríos. Modeling of protective system failures to operate for reliability studies

at the power system level using stochastic point processes.  
*International Review of Electrical Engineering*, 5(2), 2010.





