

# Reliability Assessment of Unbalanced Distribution Systems using Sequential Montecarlo Simulation

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**Abstract**—This paper explains the causes of the unbalance present in many power distribution systems and how single and double phase outages do not affect the entire load served as it is assumed in the traditional balanced analysis. A methodology based on the sequential Montecarlo simulation technique for the long term reliability assessment of these kinds of systems is presented and by means of an example it is shown how the predicted reliability indexes of an unbalanced system will be higher if its assessment is performed using balanced analysis.

**Index Terms**—Power distribution, power system reliability, power system simulation.

## I. INTRODUCTION

**D**ISTRIBUTION reliability is a field of great interest all around the world because of:

- The distribution system contributes at least with 90% of the failures that affects the power system [1], [7]. Thus, this functional zone has a great potential for the improvement of system performance and savings [1].
- Even developing countries like Colombia have established distribution reliability indexes and their upper limits. Distribution companies which overpass these limits have to pay penalties, e. g. economic compensation to affected customers [15].
- Distribution companies have to operate their systems and components to their maximum capacity and life due to economic limitations resulting from market rules imposed by de-regulation or endemic economic problems of developing countries. Resources for expansion/upgrading, replacement of aged equipment or even properly preventive maintenance are very limited and all these affect negatively service reliability.

Under these conditions, reliability assessments play a key role in the planning and decision making process of distribution companies. In order to offer credible results, these studies have to include the real constructive and operative aspects present in the systems; one of these aspects is the unbalance which is the subject of this paper.

## II. CAUSES OF UNBALANCE

Many distribution systems have been built unbalanced to reduce investment costs. Thus, single and double phase distribution transformers, secondary distribution and customer connections are extensively used. Fig. 1 shows the typical topology of unbalanced distribution feeders. They have three-phase sections and laterals with one, two or three phases.

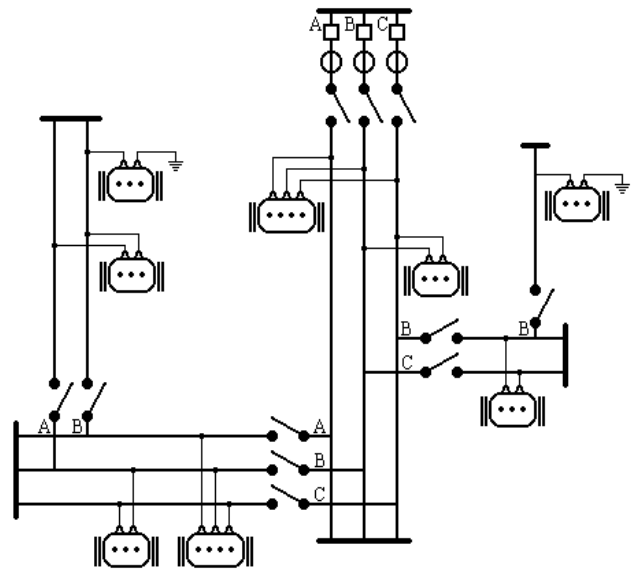


Fig 1. A typical unbalanced distribution feeder

For example, the distribution system in the city of Pereira, Colombia, has 28 three-phase radial feeders, 24 rated 13.2 kV and 4 rated 33 kV. This system has a great level of unbalance because, as it is shown in Tables I and II, 93% of customers connections are single and double phase and almost all distribution transformers are for double phase connection.

TABLE I  
CUSTOMER CONNECTIONS IN THE CITY OF PEREIRA

Low voltage connection	Customers	%
Phase - Neutral	77361	74
Phase - Phase - Neutral	21138	19
Three Phases + Neutral	8090	7
Total	106589	100

Note: year 2003

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TABLE II  
DISTRIBUTION TRANSFORMER POPULATION IN THE CITY OF PEREIRA

Rated voltage kV	Medium voltage connection		Total	%
	Phase – Phase	Three - Phase		
13.2	2150	2133	4283	99.10
33	5	34	39	0.90
Total	2155	2167	4322	100.00

Note: year 2003

Another cause of unbalance is the extensive use of single-phase distribution components such as current transformers, cut-outs, fuses, disconnectors, etc. If one of these components opens (when apply) or it fails, an unbalanced outage occurs.

Finally, other important cause of unbalance is that most failures are single-phase. In accordance to [8], 70-80% of the failures that affect power systems are single-phase and only 2-3% are three-phase.

It is important to observe that although the second and third causes mentioned before also apply for balanced distribution systems they are usually not considered for their reliability analysis.

### III. EFFECT OF OUTAGES

The number of customers affected by an outage depends on: outage type, low voltage customer connection and type of distribution transformer that supplies the customers.

#### A. Single phase outages

- Distribution transformers with phase–earth or phase–neutral connection: only the customers supplied by transformers connected to the failed phase will be affected.
- Distribution transformers with phase–phase connection: all customers will be affected.
- Three phase transformers with  $\Delta$ -y connection: The secondary voltage will be half of the normal value in two phases and normal in one phase. Thus, all customers with single phase connection to any of the phases with abnormal voltage are considered without service. For customers with double phase or three phase connection, only the load connected to the phases with abnormal voltage and all three-phase loads will be interrupted.

#### B. Double phase outages

- Distribution transformers connected phase–earth or phase–neutral: only the customers supplied by transformers connected to the failed phases will be affected.
- Distribution transformers connected phase – phase: all customers will be affected.
- Three-phase transformers with  $\Delta$ -y connection: all customers will be affected.

#### C. Three phase outages (Balanced outages)

All customers will be affected.

### IV. PROBABILITY OF OUTAGES ON FEEDER SECTIONS

When unbalance is included in the reliability assessment it is necessary to define the probability of single and double-phase failures. If a double-phase failure occurs it can be on A-B, B-C or A-C and if a single-phase failure occurs it can be on A, B or C.

These probabilities are not necessarily equal; the constructive characteristics of the feeder and the surrounding zone where the feeder is located define them. To illustrate this, let us consider the overhead three-phase feeder section shown in Fig. 2. If a single-phase failure occurs, the outage probability on phase C is higher than on phases A and B.

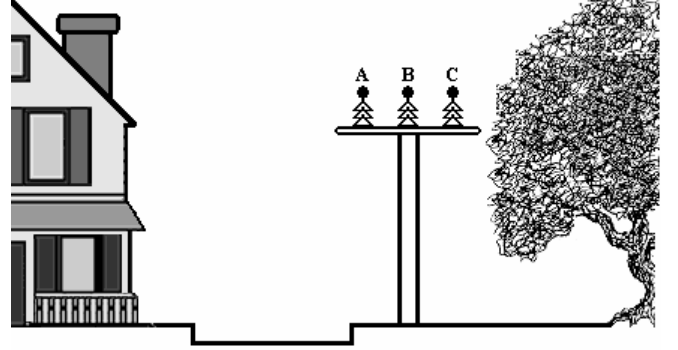


Fig 2. A Typical overhead distribution feeder

These probabilities can change during the year in accordance with seasons.

### V. METHODOLOGY

Montecarlo simulation was selected to develop the methodology for the reliability assessment of unbalanced distribution systems because it is more flexible than other techniques such as the homogeneous Markov process and the simplified method of blocks defined by a constant failure frequency ( $\lambda$ ) and a mean time to repair ( $r$ ). Its advantages are:

- It easily handles the huge sample space defined by the operative states of the components.
- It can include any kind of component probability model, not only the exponential ones.
- It can include any demand model, for example, hourly curves, probability distributions, times series, etc.
- It can include other analyzing tools such as load flow, reconfiguration, optimization techniques, etc.
- Data changes or additions are easily incorporated into the database. Software modifications are not necessary.

#### A. Component Modeling for Reliability Assessment

All system components are modeled by means of the classical two-state probability model. This kind of model considers that a component has only two states: “good” and “failed”. The transitions between states are defined by means of the probability distributions of time to failure ( $t_{ff}$ ) and time to repair ( $t_{tr}$ ).

### B. Demand Modeling

Any kind of demand model can be incorporated in the Montecarlo simulation e. g. hourly curves, probability distributions, time series, constant values, etc.

### C. Simulation Procedure

The system performance in a future year of interest is observed artificially by means of the sequential Montecarlo simulation under several scenarios defined by topology, expansion/upgrading, forecasted demand, etc.

A “simulation” is the assessment of the system performance under a specified scenario and, as shown in Fig. 3, it consists of  $N$  iterations or artificial observations of one year of system operation under the conditions of the scenario under study.

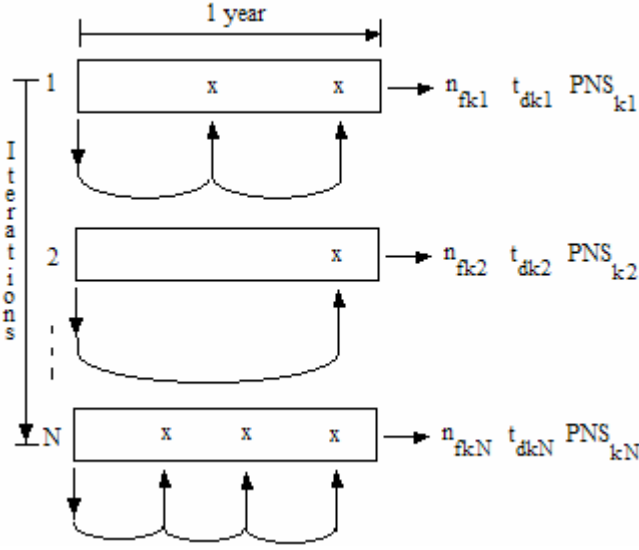


Fig 3. Simulation procedure

Within iterations, the sequence of component outages and their restorations is generated, as shown in Fig. 3, where the x's are the periods when a component is down. See Section V.D for more details of the procedure within iterations.

For a load point  $k$ , the samples of output variables such as number of failures ( $n_{fk}$ ), down time ( $t_{dk}$ ) and power not served ( $PNS_k$ ) allow the calculation of the reliability indexes. For example:

- Expected Failure Frequency

$$\lambda_k = \sum_{i=1}^N n_{fki} / N \quad (1)$$

- Mean Time to Repair

$$r_k = \sum_{i=1}^N t_{dki} / \sum_{i=1}^N n_{fki} \quad (2)$$

- Expected Load Not Served (Load Curtailed)

$$EPNS_k = \sum_{i=1}^N PNS_{ki} / N \quad (3)$$

Other reliability indexes such as SAIFI, SAIDI, etc. can also be calculated.

The output variables samples can also be fitted to probability distributions and so the probabilistic models of the reliability indexes are obtained.

Two stopping rules are used for the simulation: A fixed number of iterations and the coefficient of variation [9] of load point reliability indexes.

### D. Iteration Procedure

- Initially, the iteration time counter  $t$  is zero.
- For each component  $i$ , generate a uniform random number  $U_i$  and convert it to a time to failure  $t_{fi}$  using its distribution of time to failure.
- Obtain  $t_{tf} = \text{minimum}(t_{fi})$
- If  $(t + t_{tf}) \geq 1$  year, there is no failure and the iteration is finished.
- If  $(t + t_{tf}) < 1$ , the component with the minimum time to failure is considered “failed”.
- If the failed component is a feeder section:
  - Generate a uniform random number  $U_{ft}$  and obtain failure type as shown in Table III, where  $K_s$  is the single-phase failure probability and  $K_d$  is the cumulated probability of single-phase and double-phase failures

TABLE III  
PROCEDURE TO OBTAIN TYPE OF FAILURE

Case	Type of failure
$U_{ft} \leq K_s$	Single-phase
$K_s < U_{ft} \leq K_d$	Phase-phase
$K_d < U_{ft} \leq 1.0$	Three-phase

- Generate a uniform random number  $U_{fp}$  and obtain failed phases as shown in Table IV where  $K_1$  is the probability of failure on phase A (single phase failure case) or on phases A and B (double phase failure case) and  $K_2$  is the cumulated probability of failure on phases A and B (single phase failure case) or on phases A and B (double phase failure case).

TABLE IV  
PROCEDURE TO OBTAIN FAILED PHASES

Case	Single phase failure	Phase- phase failure
	Failed phase	Failed phases
$U_{fp} \leq K_1$	A	A and B
$K_1 < U_{fp} \leq K_2$	B	B and C
$K_2 < U_{fp} \leq 1.0$	C	C and A

- Determine the failure effect on load points: demand not served, number of affected customers.
- Generate a uniform random number  $U_r$  and convert it to a time to repair  $t_{tr}$  using the time to repair distribution of failed component.

9. For every load point  $k$  cumulate the values of the output variables: number of failures, down time, load not served, etc.
10. Simulated time after a failure  $j$  is:

$$t = t_{j-1} + t_{tf} + t_{tr} \quad (4)$$

11. If  $t < 1$  year, go to step 2, otherwise stop this iteration.

## VI. EXAMPLE

Let us consider the RBTS test system shown in Fig. 4. This test system was proposed in [5]. System data and reliability indexes are presented in [6].

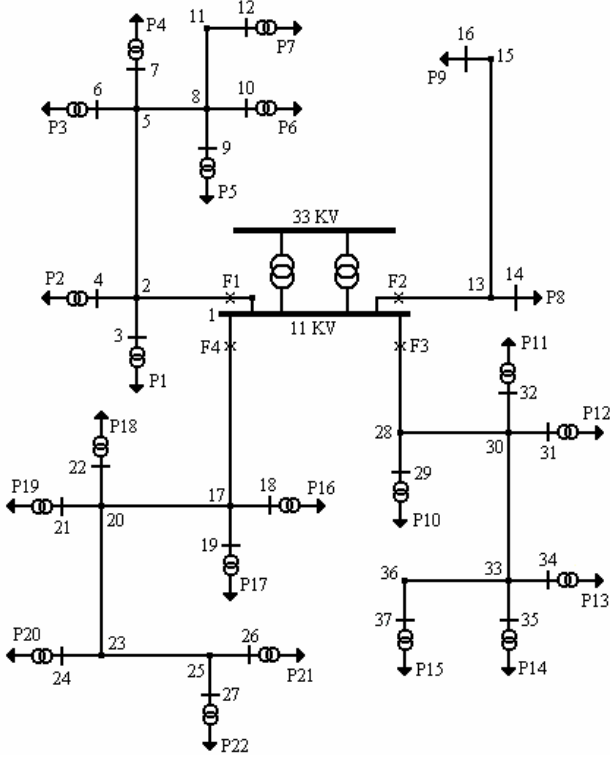


Fig 4. RBTS test system [5]

RBTS is taken as Case 1 for this example. There are two state probability models for each feeder section and distribution transformer [5].

Also, let us consider that RBTS test system is unbalanced as shown in Fig. 5. This is Case 2 for this example. Probabilities for each type of failure on three-phase sections in Case 2 are presented in Appendix A. The probabilities of failures on phases A, B, C and A-B, B-C and C-A were assumed equal.

In both cases it was assumed that feeder sections and laterals are protected by means of fuses i. e. no disconnectors were included. Also, load point demand was modeled by means of hourly active curves; thus, in accordance with [2], this kind of simulation is “full sequential”.

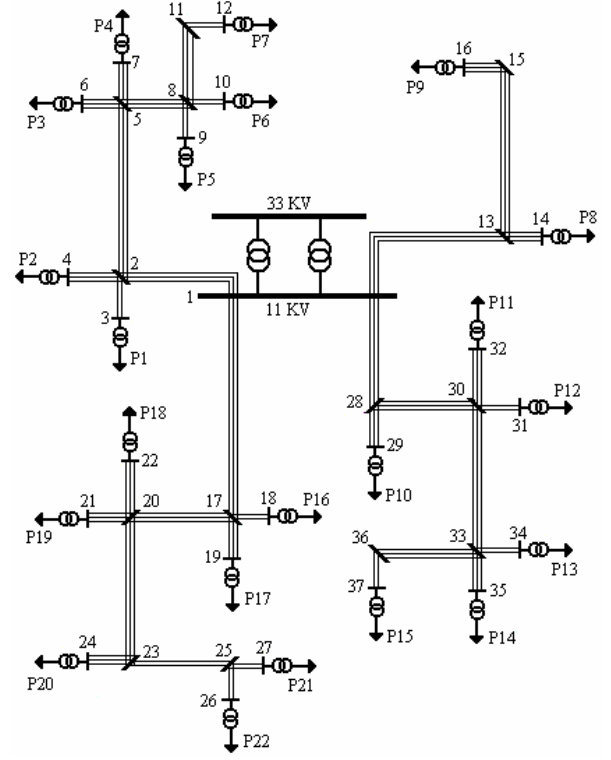


Fig 5. RBTS test system including unbalance

Tables V to VII show results. The difference between results for a given reliability index  $I$  is calculated as:

$$\Delta\% = ((I_{CASE2} - I_{CASE1}) / I_{CASE1}) * 100\% \quad (5)$$

As the reliability indexes are calculated for a future year or in the long-term, they are also called “adequacy indexes” and they all correspond to expected values.

TABLE V  
FAILURE FREQUENCY [INTERRUPTION/YEAR]

Load point	Case 1 System balanced	Case 2 System unbalanced	$\Delta\%$
1	0.0876	0.0781	-10.84
2	0.1178	0.1180	0.17
3	0.1638	0.1653	0.92
4	0.1478	0.1231	-16.71
5	0.2149	0.1812	-15.68
6	0.2134	0.1752	-17.90
7	0.2539	0.1468	-42.18
8	0.1041	0.1190	14.31
9	0.1434	0.1568	9.34
10	0.1045	0.1024	-2.01
11	0.1658	0.1324	-20.14
12	0.1694	0.1454	-14.17
13	0.2062	0.1776	-13.87
14	0.2134	0.2098	-1.69
15	0.2487	0.1428	-42.58
16	0.1156	0.1071	-7.35
17	0.1060	0.1096	3.40
18	0.1615	0.1314	-18.64
19	0.1726	0.1701	-1.45
20	0.2218	0.2198	-0.90
21	0.2522	0.2236	-11.34
22	0.2577	0.1593	-38.18

TABLE VI  
MEAN TIME TO REPAIR [HOUR/INTERRUPTION]

Load point	Case 1 System balanced	Case 2 System unbalanced	$\Delta\%$
1	5.0249	5.0954	1.40
2	5.7940	5.7306	-1.09
3	5.4532	5.5158	1.15
4	5.6160	5.5720	-0.78
5	5.3136	5.3885	1.41
6	5.3251	5.4212	1.80
7	5.2770	5.5583	5.33
8	4.9459	5.7548	16.35
9	5.0113	5.5356	10.46
10	5.9443	6.0129	1.15
11	5.4749	5.4279	-0.86
12	5.5500	5.4025	-2.66
13	5.5846	5.3467	-4.26
14	5.5440	5.4245	-2.16
15	5.4604	5.4799	0.36
16	5.7572	5.6207	-2.37
17	5.5958	5.7705	3.12
18	5.4709	5.7705	5.48
19	5.5350	5.6053	1.27
20	5.4178	5.3794	-0.71
21	5.3430	5.2678	-1.41
22	5.3498	5.5044	2.89

TABLE VII  
GLOBAL RELIABILITY INDEXES

Reliability Index	Case 1 System balanced	Case 2 System unbalanced	$\Delta\%$
SAIFI [interruptions./year]	0.1408	0.1285	-8.74
SAIDI [hours/year]	0.8998	0.8116	-9.80
CAIFI [interruptions./year]	0.0005	0.0005	0.00
CAIDI [hours/ interruption]	5.2628	5.2199	-0.82
ASAI [%]	99.9897	99.9907	0.001
ASIFI [interruption]	0.2018	0.1426	-29.34
ASIDI [hours]	1.4788	1.0278	-30.50
ENS [MWh/year]	11.0242	8.1638	-25.95

Analysis of results shows if the reliability assessment of the unbalanced test system is performed assuming it is balanced then:

- Reliability indexes based on demand such as ASIFI, ASIDI and ENS are 26 to 31% higher.
- Reliability indexes based on number of customers such as SAIFI and SAIDI are 9 - 10% higher
- Load point failure frequency is 10 - 43% higher. The load points with phase to phase connection have the higher values.
- For most of load points, there is a low difference ( $\pm 5\%$ ) in the mean time to repair.

## VII. CONCLUSIONS

- Single and double phase outages are the most frequent events that affect power system operation and they do not necessarily affect all customer demand as three-phase outages do. These important facts have to be included in the reliability assessment of three-phase balanced and unbalanced distribution systems in order to perform more realistic studies.
- As the methodology proposed in this paper shows, the sequential Montecarlo simulation is a powerful method for the reliability assessment of power distribution systems because it can easily include the constructive and operative unbalance.
- If the reliability assessment of an unbalanced distribution system assumes all outages are balanced (three-phase) the resulting reliability indexes will be higher than in the case where the unbalance is included. This kind of assessment is "pessimistic" and could falsely indicate the need of investments that are not necessary for the scenario under study.

## VIII. APPENDIX A

TABLE VIII  
PROBABILITY OF FAILURE ON FEEDER SECTIONS OF UNBALANCED TEST SYSTEM [%]

Nodes		Type of failure		
		three-phase	phase-phase	Single-phase
1	2	20	20	60
2	3	-	40	60
2	4	15	15	70
2	5	20	20	60
5	6	20	20	60
5	7	20	20	60
5	8	15	15	70
8	9	-	40	60
8	10	-	40	60
8	11	20	20	60
11	12	-	40	60
1	13	20	20	60
13	14	20	20	60
13	15	20	20	60
15	16	20	20	60
1	28	20	20	60
28	29	20	20	60
28	30	15	15	70
30	32	15	15	70
30	31	-	40	60
30	33	20	20	60
33	34	-	40	60
33	35	20	20	60
33	36	20	20	60
36	37	-	40	60
1	17	20	20	60
17	18	-	40	60
17	19	15	15	70
17	20	20	20	60
20	22	20	20	60
20	21	20	20	60
20	23	20	20	60
23	24	20	20	60
23	25	-	40	60
25	27	-	40	60
25	26	-	40	60

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## X. BIOGRAPHIES

**Carlos J. Zapata** (SM'1993, AM'1997, M'2004) was born in Cartago, Colombia, on January 25, 1966. He obtained his BScEE from Universidad Tecnológica de Pereira, Pereira, in 1991 and his MScEE from Universidad de Los Andes, Bogotá, in 1996. From 1991 to 2001 he worked for Consultoría Colombiana S. A, Bogotá, Colombia, where he participated in forty one projects in the areas of power system studies, electrical designs and software development; He was Project Manager in fifteen of these projects. In 2001, he joined Universidad Tecnológica de Pereira.

**Oscar Gómez** was born in Pereira, Colombia, on April 4, 1979. He obtained his BScEE and MScEE from Universidad Tecnológica de Pereira, Pereira, in 2003 and 2005, respectively. He worked during 2003 and 2004 in the project "Reliability Study of Regional Electric Power System" and during 2005 in the project "Methodologies for reliability assessment in power distribution companies", both sponsored by the Research and Extension Center of Universidad Tecnológica de Pereira. In 2006, he joined Universidad Tecnológica de Pereira.